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Malawi

Senior Secondary Mathematics

Form 4

Teacher's Guide

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SENIOR SECONDARY

MATHEMATICS

Teacher's Guide for Form 4

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All you need to know!

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PREFACE

This Teacher's Guide is intended to help the teacher put more emphasis on the mastery of mathematical concepts rather than on rules and simple manipulation, in order that there be greater satisfaction in the teaching and learning of mathematics. The teacher is therefore, encouraged to resist the temptation to tell his/her students, all the time, exactly what to do and how to do it. Class discussion, participation and discovery method of teaching are highly recommended. The teacher should make a deliberate effort and take time to lead the students through the concepts being learnt. In this way, there is more assurance that the student will master and retain the concepts. This book has adopted this approach as far as is possible and practical, and this guide is intended to help the teacher use the same approach for the benefit of the students.

In both the Student's Book and Teacher's Guide, the methods and approaches to various topics and problems suggested are meant to encourage and boost student's participation and to sustain their interest. However, these may not be the only ones available, and the teacher is, therefore, encouraged to think of other alternative methods and approaches.

In addition to the teaching guidelines provided, the guide also gives the teacher general information concerning the teaching of mathematics, such as scheming, planning for a lesson, and so on.

Both the students and teachers who use this guide and its Student's Book are welcome to make suggestions and recommendations which may be used to make future editions of these books even better.

GUIDE TO THE TEACHER

This Teacher's Guide has been written to be used with its Student's Book 4.

The Student's Book

The Student's Book for Form 4 comprises of 13 chapters, and each chapter has a number of sections/subtopics. Each section has a short explanation of the concept, some worked examples and an exercise.

Some of the pages have frames on them. Within these frames are highlights of basic concepts, formulas, methods, etc., that the authors felt are important. These are highlighted so that the reader can pick them out with ease, especially when revising. Bolding has often been used to highlight important facts or words. The activities are enclosed in a blue screen to distinguish them from the main text.

The worked examples in the book have all been written in italics and highlighted in blue so that they are easily distinguished from the rest of the text.

After the first five chapters, there is a set of revision exercises, based on these chapters. Another set comes after Chapter 10, based on Chapters 6 to 10. Yet another revision exercise is given after the last chapter, based on Chapters 11 and 13. This though, does not necessarily suggest sub–division into three terms' work. The authors encourage the teacher to try as much as possible to cover everything by the end of the second term in order to leave the short time available in Term 3 for examination preparation. Two sets of model papers have also been provided based on the mathematics syllabus.

The Teacher's Guide

In addition to the main parts of this guide, answers are provided for all exercises, revision exercises and model papers. The main parts of the Teacher's Guide are as follows:

Introduction

This section discusses the objectives of teaching mathematics at all levels, and in particular, the role of Mathematics in the learning of other subjects and in everyday life. It further explains how this series will help in achieving these objectives, and what methods or approaches are preferred in the book and in the teaching of secondary school mathematics.

Instructional planning

This section provides the teacher with guidance on the various 'in-teaching' and 'around-teaching' plans and activities that one may carry out. It establishes the need for having the syllabus, scheme of work, lesson plan, teaching aids and references. It also suggests what the contents of these plans may be.

Methods of teaching

This section suggests the various possible methods/approaches that can be used when implementing the plans, i.e. when teaching.

Taking care of learning difficulties

This section suggests, to the teacher, various causes of difficulties in learning, their possible symptoms and some ways of addressing them. It is not, by any means, meant to provide the complete solution. It is to act as a challenge to the teacher to start looking at those difficulties and start thinking about how to address them.

Assessment and evaluation

This section provides a discussion on assessment and evaluation, showing why this is done, how it should be done, and how the results could be analysed.

The Mathematics teacher and career development of the student

This section briefly discusses how the Mathematics teacher may guide his/her students on the importance of mathematics in their future career as a way of encouraging them to perform well in the subject.

Preparation for the examination

This section discusses how the teacher may guide his/her students in preparing for and handling the examination.

Topic-by-topic guide

This is the main part of the Teacher's Guide. It comprises of 13 chapters, corresponding to and headed in the same way as the chapters of the Student's Book. For ease of reference, each chapter has been sub–divided as follows:

(a) Background knowledge

In this section, you are simply told what the students are expected to know, or have learnt, in order for them to be able to learn the new concepts introduced in the chapter. It would be a good idea for you to read this section in advance, so that you are able to decide on whether a quick revision of the same is necessary, and hence include it in your lesson plan.

(b) Specific objectives

This is simply a statement of what is expected to have been learnt, or achieved, by the end of the chapter. The objectives are mainly derived from the syllabus. It is from there that you should derive the objectives of your lesson.

(c) Subtopics

This is the chapter breakdown.

(d) Resources

This section suggests the aids that you may need in a particular topic. This excludes the normal classroom apparatus like blackboard and chalk, of course without diminishing their importance and, indeed, their central role in aiding the learning of mathematics. Note that it is not, in any way, suggested that these are the only aids that can be used in teaching the topic. It should be a challenge to you to come up with more aids. Improvise where none is available commercially, or is unaffordable; 'improvisation' is the buzz word for a good teacher who wishes to make his/her lessons interesting to the students.

(e) Teaching guidelines

These are guidelines for teaching, i.e. how to present the concept to the students. Justifiably, the experienced teacher may want to vary the approaches used to suit his/her lessons. He/she should freely do so. In some cases, it is suggested how you may use the proposed teaching aids, while in others it is not. In all cases, it should be a challenge to you to think of the best and appropriate aids and methods to enrich your lesson.

Note that quite often, in the Student's Book, question like, 'What do you notice?', etc. are posed, or the word 'discuss' is used. This essentially means the use of question and answer technique. In some cases, the questions have been written in the Student's Book while in others, you are expected to make up your own questions. Note that heuristic and discovery are the methods of choice in this series, and so you must make sure that the students take an active part in the lessons. Remember also to organise activities/participation according to your students' needs and abilities.

(f) Answers to exercises and model papers

Note that only the final answers to questions are given. Ensure that you check that the students' working is correct and that it is systematically presented. In an examination situation, the answer will usually earn only 1 mark while the bulk of the marks comes from the working. Make sure that the students understand this.

Note

It is important that, every now and then, you do continuous assessment of your students. You can do this by coming up with questions similar to those given in the revision exercises, but only for those chapters already covered. The end of term summative test may be set in a similar way to the revision exercises given. At the end of the year, you may have a summative test, for the year, set to cover the work of the whole year.

It is very useful for the teacher to understand the correct interpretation of the secondary school Mathematics syllabus in order to be able to teach the subject effectively.

INTRODUCTION

The main objective of teaching Mathematics at all levels is to enable the student to develop clear and logical thinking, which is needed for analysing both academic and everyday life situations. Thus, Mathematics aids in understanding other subjects, especially the sciences. Apart from this, we use mathematics in everyday life more than we use any other subject.

Secondary Mathematics is designed to help students in working out solutions to problems with accuracy, precision and speed, both for academic and functional life situations. 'Mathematics is necessary for the development of scientific, technical, monetary and commercial activities around the life of an individual and the community.' *(H.O. Ayot and M. M. Patel, Instructional methods, Educational Research and Publications (ERAO), (1992))*

Appropriate knowledge in Mathematics helps the student to improve his/her skills in measuring, approximating and estimating. Such skills are necessary for any quest, be it academic, business, or whatever. It also aids him/her in collecting, representing, and interpreting data which he/she can manipulate and add meaning to. Mathematics, therefore, helps the student to develop investigative and problem solving skills, thus enabling him/her to better understand and manage his/her personal and collective life.

Due to the foregoing, we see that the teaching of Mathematics must be very carefully approached as it could affect the students' performance, not only in Mathematics itself, but also in other subjects as well as in other activities in life. To be successful, students must grasp the essential computational skills and concepts.

This series structures, sequences and develops Mathematical experiences through concrete activities in order to help students acquire the much needed computational skills. It is written with the student in mind, towards whom instruction and learning is geared. Remember that as we plan, prepare and present content and instructions, it is the student who must learn. If at the end of it all the student has not learnt, then we have not achieved our objectives as teachers.

In the Student's Book and in this Teacher's Guide, various methodologies have been suggested to suit the student's learning at various stages. Since learning is better and long lasting if it is by doing, student–centred activities have been emphasised. It is to be appreciated that it is difficult for the authors to come up with all possible activities and resources. The teacher is, therefore, challenged to come up with others, that will be better suited to his/her particular environment. Activities that involve students in handling and manipulating materials will enable them to make important discoveries and gain sufficient practice to allow them concretise and master various concepts.

A concept is a structure which imprints in the mind of the student various quantities and objects. When one learns, for example, about fractions, he/she appreciates their form and properties that make them different from other numbers.

Mathematics is concerned with patterns or structures which have to be pictured and manipulated in the mind, hence the common reference to Mathematics as being abstract. The mental pictures have to be created through many experiences of things, handling them, observing them, constructing imaginative representations of them, symbolising, sequencing and so on.

Mathematical concept formation generally arises from three types of experiences:

1. Repetition of experiences of the same thing or event.
2. Contrasting between two or more different things or events.
3. Manipulation of things and/or observation of their behaviour.

For this reason, the authors emphasise on practice through various student–oriented activities and exercises. The teacher is advised to device more activities using appropriate teaching aids.

It is very useful for the teacher to understand the correct interpretation of the secondary school Mathematics syllabus in order to be able to teach the subject effectively. For this reason, the authors' interpretation of the syllabus, is presented.

INSTRUCTIONAL PLANNING AND METHODS OF TEACHING

INSTRUCTIONAL PLANNING

Planning is very essential in anything we do, and teaching is no exception. In the teaching of Mathematics, planning is very important and is a major factor in attaining effectiveness. There are three levels of planning that are pertinent to effective teaching - the syllabus, the scheme of work and the lesson plan.

Syllabus

The first level plan i.e. the syllabus is provided by the Malawi Institute of Education. It is important that a Mathematics teacher takes time to understand the contents and objectives stated therein. The Mathematics syllabus outlines what needs to be covered at each level of learning. It spells out objectives to be achieved in each topic. The syllabus has been organised in the order the topics may be taught. However, the teacher has the discretion to teach them in the order he/she deems most logical. The teacher is expected to work out details of what is to be taught in each lesson.

Scheme of work

A scheme of work is a carefully worked-out plan of action. It quantifies how much can be taught in one term. The scheme could show the number of lessons each subtopic would need or it could show what is to be taught in each lesson. The scheme of work, if properly done, helps teachers to allocate appropriate time durations to the various subtopics.

It is worth noting that there are different formats of schemes of work. Different institutions use formats that they have agreed on. The following format shows the aspects which should be included in a scheme of work.

Since students do not learn at the same pace, some may be slow while others may be fast, it is possible to have different schemes of work for different same year classes in the same school.

Lesson plan

If you are to make a long journey, there are certain things you will do in preparation for the journey. You will have a reason (purpose or objective) for travelling. You will carry some item(s) to assist you in the journey as well as some contacts of places or people. Similarly, planning a lesson is equally involving. Teaching without lesson planning is like going on a journey without planning or making any preparation. So, good teachers always prepare and plan their lessons even if it is a subtopic they have taught before. A lesson plan is extracted from the scheme of work and contains the same elements as in the scheme of work. The objectives in the lesson plan should be clearly and precisely stated. The various steps of the lesson should be elaborate enough showing the details for each step and each activity in the lesson. Teaching aids and references should be included in the lesson plan. A lesson plan has the following parts:

1. Subtopic or title of the lesson
This corresponds to a subtopic(s) as given in the syllabus.

Week	*Lesson no.*	*Topic and subtopic*	*Objectives*	*Teaching*	*Reference aids*	*Remarks, date taught*

A sample scheme o fwork

2. Objectives
Objectives in a lesson plan should be stated clearly in behavioural and measurable terms. For example;
By the end of the lesson, the students should be able to
(a) state …
(b) define …
(c) convert …
(d) add …
(c) calculate.

3. Teaching aids/Resources
Teaching aids are necessary for good teaching. Good teaching aids, when used appropriately, make teaching easy and students are able to grasp the concepts much more easily than if the teaching aids were not used. If a

wrong or inappropriate aid is used, it could block learning instead of aiding it. A good teaching aid should be of reasonable size so that it can be seen by the whole class without any problem. It should be relevant to the content. A good teacher should be able to improvise appropriate teaching aids.

4. References

References could include textbooks, magazines, past examination papers, teacher's notes, etc. Note that for textbooks (and magazines), you should state the name/title of the textbook (or magazine), name(s) of author(s) and page numbers where the content is found.

5. Lesson presentation

Lesson presentation has four major steps:

(a) Introduction

This is the stage of the lesson where the teacher captures the interest of the students. Students are helped to focus on the main theme of the lesson. Introduction should always be brief, not lasting more than 5 minutes.

(b) Development of the lesson

This part of the lesson shows

(i) the content to be learnt,
(ii) the teacher's activities in helping the learning process,
(iii) students' activities/involvement in the learning process,
(iv) the method(s) used and any teaching aids used in the lesson.

(c) Consolidation

This is a session (also called supervised practice) where students put in practice, individually, what they have learned, by solving some selected problem(s) as the teacher attends to those who might have difficulties.

(d) Conclusion/summary

The teacher and the students summarise what has been learnt and assess how well objectives have been achieved. Homework/assignment is usually given at this point.

Record of work

This is a record kept on a daily or weekly basis. It indicates the topics/subtopics taught in each class. Keeping this record helps the teacher keep track of coverage of the syllabus as per the scheme of work. It would also help any other teacher know what a given class has covered and where to begin should there be need for another teacher to take over the class.

METHODS OF TEACHING

Some of the methods commonly used in teaching Mathematics include:

(a) Lecture method
(b) Heuristic method (Questioning technique)
(c) Problem solving
(d) Activity - based method
    (i) Group work
    (ii) Individual work
(e) Discussion method

It is strongly recommended that whichever method one chooses to teach a Mathematics lesson, one should plan

(i) the activities which will help students grasp the topic,
(ii) to have the lesson as student-centred,
(iii) the experiments that would be done, and
(iv) the improvisation that would help maximise the learning process.

A Chinese saying states:

I hear and I forget
I see and I remember
I do and I understand.

With this saying in mind, let us now look at the various methods of teaching and decide on the most appropriate one at this level.

(a) Lecture method

Lecture method requires people who have no problem with the language used and can make notes as the lecture progresses. It is known that many students are not really competent in English, the medium of communication, and so cannot make good notes. As the Chinese saying goes 'I hear and I forget … ', students will learn very little by lecture method. This method is too teacher-centred and is not recommended at this level, especially in teaching Mathematics.

(b) Heuristic method (questioning technique)
This method is very much encouraged. However, it requires a good mode of questioning if one is to achieve the desired results. When well used in a lesson, it

(i) increases students' participation,
(ii) creates enthusiasm and motivation in the student,
(iii) encourages creative thinking,
(iv) develops skills in organisation of ideas,
(v) promotes interaction between the teacher and the students.

To be effective, the teacher should ask probing questions. The questions should be prompting such that students will get hints and clues in order to arrive at the expected response. Sound planning is required of the teacher in order to have effective class participation.

(c) Problem solving method
The teacher needs to help students grasp the process of solving mathematical problems. One has to

(i) understand the problem posed,
(ii) note down all the information given and sort out other information and operations needed to solve the problem,
(iii) systematically solve the problem,
(iv) check that the solution is realistic.

(d) Activity-based method
(i) Group work
Group work enhances the spirit of working together and active participation by all. Students gain confidence in discussing, organising and carrying out activities. The teacher is expected to plan and organise the group and group activities and also check the progress of all groups, and give guidance to each group according to the needs arising.

(ii) Individual work
In this case, each student carries out an activity under the guidance and supervision of the teacher. This method is advantageous because no student can ride on the back of the others as can happen in group work.

(e) Discussion method

The teacher acts as a facilitator of the discussion while students pose questions and others answer them. Positive and relevant ideas are encouraged while irrelevant ideas are discouraged by the teacher.
Through discussions, students should learn how to apply knowledge, think critically, solve problems and make decisions. All members of a discussion group/class should be actively involved and should be given an opportunity to make their contribution.

Conclusion
Most Mathematics lessons could be effectively taught using such methods to varying degrees. Whichever way a lesson be conducted, it is important that the lesson be student-centred. Students should be most involved in 'doing'. As the Chinese saying goes 'I do and I understand'. This far, the success factors for a lesson may be summarised in the abbreviation PDSI, which is short for Plan, Do, See and Improve. Explained briefly, this means:

Plan
You must take time to plan all the appropriate activities, methods and teaching aids that you will need for effective learning to take place.

Do
Implement your plan, ensuring maximum doing, i.e. participation by the students in the lesson. Students' participation is most important and should be through what may be called 'hands-on, hearts-on, minds-on, mouths-on and eyes-on' activities.

See
You should see (observe) and get all feedback of the lesson to help assess the success or failure of the lesson.

Improve
You should, all the time, be open to note anything that would help improve teaching or learning. Thus, in a lesson, all activities should, as much as possible, be student-centred rather than teacher-centred.

TAKING CARE OF LEARNING DIFFICULTIES

It is important that every teacher knows his/her students well. He/she should know those with difficulties in learning; the extent, ranging from mild to severe; and seek to understand the cause(s) of the difficulties and ways to mitigate them.

For example, some students may have difficulties or be completely incapable of performing some physical activities like ‘pacing out steps on a number line’. In such a case, the teacher needs to direct and guide such a student to perform the activity in other ways, e.g. with the hands using a table-top board which has been graduate appropriately to provide the steps for the required movement.

Other than physical disabilities like impared limbs, blindness, deafness and dumbness, there are many other causes of learning difficulties that many teachers hardly think about. The following are examples of such causes, likely symptoms, and some suggestions on how to address them.

Cause	*Likely symptoms*	*Suggested action*
Poor attitude towards mathematics (most prevalent)	• Indifference. • Aggressiveness. • Failure to do homework. • Doing other things during the lesson. • Shoddy work. • Outright talk of disliking Mathematics or of the subject being difficult etc.	• Use activities that make learning of mathematics more interesting and more engaging. • Use of simple approaches when teaching concepts. • Link mathematics with other subjects, real life situations and various careers, etc. in order to show its importance. • Counselling.
Abnormal mental ability, i.e. specially gifted or slow students	**Specially gifted** • Completing tasks too fast. • Working ahead of schedule. • Getting bored too fast. • Easily distracted. • Hyperactivity. • Truancy with lots of creativity/ingenuity. **Slow student** • Taking too long to complete tasks. • Giving up too soon. • Rarely getting tasks right even when they complete them. • Shoddy work. • Frustration.	• Give graded work to ensure that the specially gifted students have enough challenge and the slow students have tasks that they are able to perform for them to be motivated. • Give more individualised attention to slow students, even if it means sometimes going out of your scheduled hours. • Talk to parents and recommend remedial learning to enhance concept building and enable the slow students to catch up with others.

Low vision and colour blindness	• Squinting and blinking. • Reading from other students' work instead of from the chalkboard or chart. • Weeping/wet eyes. • Frustration. • Seeing wrong colours. • Failure to distinguish objects from their backgrounds.	• Encourage the student to sit nearer the board. • Confirm often that students can read what is on the chalkboard or chart. • Use larger handwriting on the board. • Use visual aids such as charts that are large enough. • Ensure good colour contrast on learning aids (i.e. avoiding using colour shades that are close to each other). • Talk to parents or guardians and recommend medical attention.
Poor hearing	• Cocking of the head. • Regular consultation with their deskmates. • Complaints like, 'we can't hear you'. • Delayed responses unless the student is looking at you. • Irrelevant responses. • Frustration. • Speaking too loudly. • Cupping the ear in an attempt to hear you better.	• Ensure you are loud enough, without shouting. • Encourage the student to sit closer to the front. • Regularly check students' work in their exercise books to ensure they put down the right things, i.e. they heard you right. • Sometimes have them repeat what you say, without drawing any undue attention to individuals. • Talk to parents and recommend medical help.
Sickness (usually a temporary cause)	• Lack of concentration. • Drowsiness and restlessness. • Blood shot eyes. • Being unkempt. • Sluggishness, etc.	• Give first aid where possible. • Advice their parents to seek medical attention.
Shyness/introvertedness/ low self esteem	• Unwillingness to participate in class as an individual or even in a group. • Unwillingness to be at the front. • Not able to look at others in the face. • Nail biting. • Looking embarassed when or after answering a question in class.	• Encourage and give them general counselling. • Use activities where everybody has a role to play. • Help boost student's self-esteem by focusing more on his/her positive qualities than negative ones. • Encourage participation in games.

Speech difficulties	• Stammering or stuttering. • Lisping and shrubbing, thus becoming embarrassed, e.g. 'hex' for 'x', 'hand five' for 'add five', 'sousand' for 'thousand', 'dorry' for 'sorry, 'patton' for 'pardon', etc.	• Put the student at ease by making him/her understand that everyone has his/her weaknesses that are not of their own making. • Encourage all to laugh about their own weaknesses even as they make efforts to overcome them.
Language difficulties	• Poor grammar. • Unwillingness to speak in English. • Inability to express oneself. • Sketchy work. • Copying questions and not solving them.	• Encourage the student to take remedial lessons in the language. • Encourage the student to speak more in the language. • Encourage the student to read novels, magazines, etc.
Social factors • Traumas like death or sickness in family, family breakup, etc. • Poor family economy leading to lack of fees, want for clothes and shelter, hunger, etc. • Peer misguidance. • Poor boy/girl relationship as well as adolescence maladjustment. • Difficult childhood, etc.	• Withdrawal, breaking down, being unkempt, anger, sudden drop in performance, lateness to school or to classes, skipping lessons or days, reduced interest not only in learning but also in what happens around. • Behaviour not comensurate with student's age (e.g. childishness). • Seeking attention. • Behaving out of character.	• Counselling and rehabilitation. • Encouraging socialisation with others less affected. • Encouraging the student to talk about it, at the same time assuring him/her of confidentiality. • Encourage participation in games/sports/exercises/play as a way of distracting them from the problem. • Encourage spirituality yet being careful not to lead the student to fanaticism. • Talking to parents/guardians.
Drugs/substance abuse	• Withdrawal, drooping mouth and eyes, mood swings, sluggish speech, drop in performance, being unkempt, red/blood shot eyes, missing classes, loss of interest in learning and what happens around, destructiveness, general aggression, petty crimes (e.g. stealing other students' money and other items), truancy, paranoia, etc. • Suicidal tendency.	• Establish rapport with the student in order to encourage him/her to share and accept that he/she has a problem. • Seek guidance from those with expertise in this area. • Recommend the student for counselling and rehabilitation. • Involve parents/guardians in seeking solutions to the problem.

Note that these problems are not unique to students. A teacher may also be affected by similar problems and be unable to perform his/her teaching responsibilities effectively. For this reason, a teacher should try to understand himself/herself in regard to these challanges and make efforts to overcome them.

As you read this section, the authors would like you to appreciate the fact that they have given these suggestions mainly from their experience and reasoning. They have attempted to give you what they consider as some of the obvious causes of poor performance in school, symptoms and possible actions that can be taken. Thus, they would like you to bear in mind that the situation could be more complicated and so demand that the student be referred to an expert. You should therefore seek to know who the experts are, within the school or in the locality, to whom you can refer the difficult cases.

ASSESSMENT AND EVALUATION

Introduction

Assessment is a very important component of teaching. Assessment is done so as to gather information that is used by the teacher and students or by the society, e.g. parents, employers, etc. In school, the assessment that takes place is **diagnostic.** It reveals errors made by students. It gives insight into the type of concepts students have and the methods they use in solving problems.

Assessment and evaluation help to check if objectives set out for the teaching of the content have been achieved. Thus objectives, content and assessment/evaluation are linked up like a three-legged stool which cannot stand when one component is missing. Thus without assessment, teaching is incomplete and is of little value.

When is assessment done?

(a) During teaching

The teacher should be able to carry out assessment at all times, and hence be able to tell when the students have mastered the concept and skills at a given stage in order to be able to move on to the next stage. This assessment can be done through

(i) observation of students' non–verbal behaviour,
(ii) oral questions, or
(iii) written work.

The teacher assesses the students as well as himself/herself in the light of students' work.

(b) After a topic(s) has been taught

This is done by having a quiz, which tests various aspects of the topic/subtopics. Such quizzes can be held at regular intervals, e.g. monthly, and hence form a continuous assessment thus providing **formative evaluation.**

(c) At the end of the year

Examinations held at the end of the year cover work done during the year. Such examinations, which are also used to rank students, are a form of

summative assessment.

Different ways of assessing

Assessment has to be done continuously. For this to be effective and meaningful there is need to use various methods as follows.

(a) Observation

It is important that a teacher allows time during the lesson to observe how students work out mathematical problems. For example, if students are doing some geometrical construction, the teacher would be able to assess and see students who might have misunderstood or mistaken the required steps.

(b) Oral

It is, at times, necessary to encourage students to discuss and explain what they are doing, how they are doing it and why they are doing it that way. Once the teacher knows the reasoning a student has for doing what he/she is doing, it will be easier for the teacher to help the student.

(c) Assignments

Assignments are given after a lesson as a way of reinforcing what has been taught, giving further practice, as well as helping to identify possible problem areas. It is important that students' assignments be marked and marks be awarded and appropriate comments made.

(d) Quizzes/Tests

Once a topic has been taught, students would be expected to do a number of questions within a limited time.

Testing is done with the aim of

(i) getting feedback on realisation of objectives and appropriateness of methods used,

(ii) identifying any weaknesses in the teaching and seeking to find corrective measures that would help improve the learning,

(iii) helping the teacher make decisions such as what the next move should be, e.g. should the class start a new topic/subtopic?, might there be students who need special attention?, etc.

A good test

A good test should have the following characteristics:

(a) Clarity — There should be no ambiguity and no possible misinterpretation.

(b) Simplicity — The items are aimed at the objectives covered without being unnecessarily difficult.

(c) Specificity — The test items must be specific to the learning levels spelt out in each item.

(d) Challenge — The items set must cover the content well enough and be able to discriminate.

How to set a test:

(a) Identify objectives to be measured by the test.

(b) Define the objectives in specific measurable terms.

(c) Outline the content matter to be tested.

(d) Prepare a table of specifications, as below, where a, b, c, d, etc. are the subtopics (content) to be tested.
Decide on the number of questions for each subtopic and the level to be tested.

Objectives / *Contents*	*Knowledge*	*Comprehension*	*Application*	*Analysis*	*Total no. of qns.*
a	√	√			
b			√		
c	√				
d				√	
. . . .					
Total no. of qns.					

(e) Construct test items from the table of specifications.

(f) Draw a marking scheme for the test. The marking scheme should show clearly where to award marks and for what purpose.

Progress record chart

A chart for recording students' progress should be drawn early in the term. It should show the various assignments (A_1, A_2, …), tests (T_1, T_2, …), end term exam and the final mark for the term as shown below.

Dates									
Test No. & topic / Names	A_1	T_1	A_2	T_2	A_3	T_3	End term exam	Final mark	Re-marks
	$\frac{x}{100}$	$\frac{x}{100}$	$\frac{x}{100}$	$\frac{x}{100}$	$\frac{x}{100}$	$\frac{x}{100}$	$\frac{x}{100}$		

The table could also show the mean mark of the class for each component and the standard deviation.

From the table one would be able to tell the performance of any given student, whether he/she is improving or not, etc.

THE MATHEMATICS TEACHER AND CAREER DEVELOPMENT OF THE STUDENT

You, as a teacher of Mathematics, are, hopefully, concerned with the performance of your students in the final exam. There is also the future career progression of the student that you have to think about. So you must give guidance, to the student, towards achieving his/her career desires.

How do you help the student?

You must continue to show the student the importance of Mathematics virtually in every career. There are certain things that you should do, that will help the student in the right direction. The following is a list, probably not exhaustive, of these things;

- Help the student develop self-esteem and look at him/herself more positively, i.e. believe in him/herself or in his/her abilities.
- Help the student identify what his/her interests are and what he/she would like to do; identify and focus on his/her talents/strengths; identify and accept his/her weaknesses and limitations and be able to see this awareness as a learning experience so as to be able to mitigate them.
- Help the student learn to respect hard work as the major contributing factor to success; no shortcuts.
- Teach the student how to manage time: Insist on eight hours of sleep and full utilisation of daytime hours: Insist that the student makes a schedule of work and balance between various activities.
- Encourage the student to develop the art/skill of concentration (in order to avoid having to waste too much time later trying to learn what he/she should have learnt in class). This calls for self discipline.
- Help the student overcome stereotypes about what they can or cannot do, e.g. the general belief that Mathematics is hard, and especially the attitude by many girls that Mathematics (and the sciences) is a preserve for boys.
- Encourage the student to seek help whenever in difficult situation and not to give up, be open minded in order to learn from others.
- Help the student learn to identify mistakes and correct them. He/she should see mistakes as opportunities to learn and improve him/herself.

At the same time, he/she should not dwell on the problem (doing so will make it grow bigger) but on the solution.

- Encourage the student to do things for him/herself (and not merely do them to please others). Once you do things for yourself, those who have expectations from you may also be satisfied. Do not depend on others for drive and motivation. You are the one who matters. The rest may share in your happiness.
- Encourage friendship, especially with people who have a positive attitude towards life.

So, whose career?

Of course, as a teacher, you already have a career! Using this fact, impress on the student that it is his/her future that you are talking about and not yours. Neither is it his/her parents or anybody else's for that matter. As you do this, you should be aiming at helping the student acquire the right attitude, i.e. that which will enable him/her achieve his/her dreams and aspirations. Enlighten the student about the basic attitude and esteem-building principles that he/she must strive to achieve. These are as follows:

- Be willing to receive honest and useful feedback: Listen to what others say/think about you.
- Know your weaknesses and limitations.
- Know what your interests are. Remember that the career is yours, not your parents', your teachers', or any other persons'.
- Have a dream of what you would like to be.
- Focus on and plan what you want to achieve.
- Look for role models whom you can emulate.
- Refuse to be discouraged or derailed by other people. The way may be rough, but there is always light at the end of the tunnel.
- Believe in yourself: You can do anything that you set your mind to do so long as you set realistic and attainable goals.
- Whenever possible and whenever available, read profiles of successful people: They will certainly inspire you, as they illustrate to you how difficult situations can be overcome. According to a renowned neurosurgeon by the name Dr. Ben Carson, the key to success is **"Think Big",** which he interprets as follows:

  Talent — identify and recognise your talent, and/or

  Time — use and manage your time wisely.

Hope — anticipate that good things will happen, and/or
Honesty — this makes one's world believable.
Insight — learn from those who are where you want to go (role models, successful people) and avoid repeating their mistakes.
Nice — be nice to all people (including yourself).
Knowledge — seek knowledge in a particular field. This will make you an invaluable asset.
Books — read many books to broaden your knowledge and develop your mind.
In-depth learning — make knowledge gained a part of you and avoid cramming.
God — He is important and holds everything together: Pray and trust in Him.

- Have self-determination and always work hard and unrelentlessly on whatever you undertake to do.
- Always be open (-minded) to learn from others and from different situations.

It would be a good idea to have the student write down these basic principles and commit as many of them to memory as possible.

Now, what is the role of Mathematics in the student's career?
You must put into focus the central role that Mathematics (or simply stated, numeracy) plays in most (if not all) careers that the student may think of. Though you are a teacher of Mathematics (and probably not one for careers guidance), take some time to enlighten the student on the central role of Mathematics in most careers. For example, you may cite the following cases (or others that you know).

Careers in Mathematics	
• Medicine, • Architecture • Engineering • Surveying • Graphic Design	• Banking • Accounting • Acturial science • Economics • Teaching

Emphasise to the student that

(i) those who are good in Mathematics should aim at excelling in the subject so that they become applicants of the subject in various fields, and

(ii) those who are not well gifted in the subject should work hard enough to become reasonably numerate so that they may be trainable in the areas of their interest, as basic mathematical skills will be required all too often.

Let the student know that even for some careers/courses which appear as if they do not require mathematics, it will actually be required at one stage or another. For instance, a historian who wants to do an advanced degree course in his/her subject area must do some mathematics in order to be able to do research work. Similarly, a person doing advanced cookery must be good in estimation, working with proportions, budgeting, and so on.

If you spared time to discuss these things with your students, you will have done a lot of good for the future careers of the majority.

PREPARATION FOR THE EXAMINATION

An extra chapter, on model papers, on exam preparation is included in the Student's Book in order to help the students in preparing for the senior certificate examination. The authors have assumed that you will have covered the syllabus early enough (preferably by the end of the second term of Form 4) and therefore there is enough time for revision. Even as you do revision in your own preferred style, it is important that you emphasise the need to show all the working and of neat layout of the solutions. Let the students know that the bulk of the marks are earned through the working and not by the final answer, which may earn only one mark.

Two sets of model papers are provided. Take them through one set, remembering to emphasise on timing for each question. You could use the other set for "mock examination" practice to test the students "against the clock".

As the date of the examination approaches, it is important that you remind students of some things that might sound obvious yet can have serious consequences if ignored. Remind them that before the actual day of the examination, they should make sure that they know:

- The date, day, time and venue of each paper.
- How to get to the examination venue without getting late.
- Their candidate numbers.
- Their examination centre number.
- The telephone number of the examination centre, if available and if necessary.

Also remind the students to prepare, in advance, any equipment that they will need for the examination. Such equipment include:

- Pens which are comfortable and reliable to use.
- Sharp pencils, pencil sharpeners and rubbers.
- Drawing instruments (preferably full geometrical set).
- Books of mathematical tables and calculators.
- Accurate watches or small clocks.

Remind your students of the importance of reading instructions carefully. When the invigilator signals that the examination may begin, they should start by reading the instructions on the examination paper very carefully. They should also make sure that it is the correct examination!

Ensure the students note the following:

- The number of sections and questions that they are required to do.
- The amount of time allocated for each paper.
- Which questions (if any) are compulsory.
- How to present the answers, e.g. in spaces provided on the question paper, or on answer booklets, etc.

Before the day of the examination, you should have drilled your students on quick calculation of the length of time they should spend on each question. You should have helped them do this using past papers, but reminding them that they use instructions on the actual examination paper. They should try to allow some 10 minutes at the end for checking their papers.

Advice your students that they should first read through the whole paper once the invigilator has indicated that it is time to start. They should do so carefully, checking that they have read every page.

If the candidate has to choose questions, he or she should:

- Cross the ones he or she cannot do.
- Tick those he or she cannot do.
- Choose the correct number to do.
- Mark the order in which to attempt them, **attempting one's best question(s) first.**

Tell your students to answer full questions if they can, but let them know that they can sometimes pass an examination by answering a lot of part questions. In questions that are structured, the first part is usually easier to answer than later parts. In such questions, marks to be awarded to each part are usually indicated. This should help the candidate to decide which parts to do.

Emphasise to your students that before attempting to answer a question, they should read it all over again carefully, jotting down points such as formulas and information relating to that question. Drill them the following hints which should help them when writing an answer:

- Ensure that handwriting is legible.
- Draw large and clearly labelled diagrams whenever appropriate.
- Present the solution in a neat and logically flowing manner, without adding any unnecessary material.

- Show all the working as most of the marks are given for working and not the answers.
- Solve the problem set and not the one you think should have been set.
- Do not do things that you have not been asked to do e.g. do not do proofs unless you are specifically asked to.
- State any principle, formulas, etc. used and indicate the reasons for using them.
- Check any formula used with the book of mathematical tables and formulas allowed into the examination room.
- Use and state the correct units.
- Always do a rough estimate of any calculation to check that the answer is sensible.
- When using a calculator, ensure that all steps of the working are shown on the answer sheet.
- State the answer to the required degree of accuracy.
- When a question says “hence or otherwise”, try “hence” first since it is usually easier and use suggestions given in the first part of the question.
- If you get “stuck”, re-read the question carefully to see if you have missed any important information or hints given in the question.
- Once the solution is complete, re-read the question against the solution, to see that you have answered all the parts.

Remind your students that, as part of examination discipline, they should try to keep to the times they have allocated to answering questions and that they answer the correct number of questions. If one answers less than the required number, they limit the number of marks available to them.

If, in any question, one cannot see how to solve a problem fairly quickly, he or she should leave it and return to it later if he/she has time: A fresh look at a question often helps. It is important to note that one should not overrun his/her time allocation to a question by more than a minute or so. One should not be lured by the thought, “Just a few more minutes and I will have the answer”. Thus, as mentioned earlier, one should answer the easier parts of the questions first (usually the first parts) and leave the rest if the solution is not forthcoming, in order to ensure that he/she attempts all the questions required.

Finally, you must re-emphasise to your students that before handing in the examination papers, they should check that the "front page" is completed according to the instructions and that any loose sheet used is clearly marked with the candidates examination number.

1 MATRICES

(Student's Book pages 1–10)

Background knowledge

Basic operations on numbers.

Objectives

By the end of this chapter, the student should be able to:

(a) present information in a matrix form,
(b) identify the order of matrix,
(c) identify square matrix,
(d) identify row matrix,
(e) identify column matrix,
(f) identify zero matrix,
(g) locate elements of a matrix,
(h) add matrices together when possible,
(i) subtract matrices when possible,
(j) multiply a matrix by a scalar (number),
(k) multiply a matrix by another matrix of order 2.

Subtopics

- Matrix.
- Order of matrix.
- Addition and Subtraction of matrices.
- Compatibility in addition or subtraction.
- Scalar multiplication of matrices.
- Multiplication of matrices
  - Multiplication of row and column matrices.
  - General matrix multiplication.
- Identity and zero matrices.

Resources

- Chart showing rectangular arrangement of information.
- Charts illustrating compatibility of matrices
- Calculators.

Teaching guidelines

Matrix

Present to the students, say on a chart, information arranged in the form of a table, such as Table 1.1 in the Student's Book. Using this, help students understand what a matrix is.

Point out to students that matrices are shown using either curved or square brackets, and talk about the various denotations of matrices, i.e. bolded capital letter when printed or a capital letter with a wavy underline when handwritten. Talk about the term "element" of a matrix.

Order of a matrix

Define the order of a matrix as "the number of rows by the number of columns", written as "$m \times n$" and read as "*m* by *n*". Point out to the students that in the context of matrices, $m \times n$ does not mean multiplication as would be the case in arithmetic.

Let students give examples of matrices of various orders.

Define row, column and square matrices and let students give examples.

Though not specifically included in the syllabus, mention the denotation $a_{m,n}$ which means the element in the *m*th row and *n*th column of the matrix **A.** Point out to the students that the subscripts must be whole numbers. Rational numbers cannot be used, so that $a_{1.2,\ 3.4}$ has no meaning in the context of matrices.

Now ask students to do Exercise 1.1.

Addition and subtraction of matrices

Using everyday experiences, such as the one in Table 1.3, in the Student's Book, illustrate the process of addition of matrices. Subtraction follows the same pattern as addition. Note that the syllabus does not indicate that we teach subtraction but it is necessary to include it.

Highlight the "same order" condition for the compatibility of matrices in addition.

Using Example 1.1, or a similar one, illustrate the fact that matrix addition is commutative while subtraction is not. Let the students do Exercise 1.2, ensuring that you check to see what answers the students give for Question 3(c). It should be “associative”. Confirm that students understand what it means.

Scalar multiplication of matrices

Using repeated addition of matrices, explain the process of scalar multiplication of a matrix.

Note that we often encounter equations involving matrices, which require the use of scalar multiplication and the idea of equality of matrices. Example 1.2 illustrates this. Highlight the condition for equality of two matrices, i.e. “same order and corresponding elements equal”.

Ask students to do Exercise 1.3.

Multiplication of matrices

Multiplication of row and column matrices

The procedure is best illustrated using everyday experiences, such as Example 1.3 in the Student's Book. What to highlight here is that the number of elements in the row matrix and the column matrix must be the same and the corresponding elements are multiplied together before adding the results. Example 1.4 further illustrates this process.

Ask students to do Exercise 1.4.

General matrix multiplication

This is an extension of multiplication of row and column matrices. As in Example 1.5, we take one row and one column at a time, and combine them. Guide students through Example 1.6 in order to master the procedure, and make sure that they are able to tell the row and column of the operand matrices from which an element in the product matrix was obtained (See Table 1.5). Check this using Example 1.7, and any other example of your choice.

Compatibility in multiplication

Give two matrices **P** and **Q** of different orders and ask students to work out the two products **PQ** and **QP** and decide which one is possible. This way it will be easy to discover the condition for compatibility in multiplication. This is highlighted in the Student's Book.

Using Example 1.8, or a similar one, guide students to discover that matrix multiplication is not commutative. The students should now do Exercise 1.5.

Check Questions 7 and 8 to see that the students make the required deductions, i.e. "Matrix multiplication is associative" and "Matrix multiplication is distributive over addition".

Identity and zero matrices

Using results of Exercise 1.5, Question 6 (a), (b) and (e), let the students observe that a matrix multiplied by such a matrix as $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, etc. does not change. Give more examples for the students to see that it is indeed so. Let the students pre-multiply and post-multiply by such matrices and see that the order in which the matrices are multiplied does not really matter, provided that both matrices are square matrices!

Now define the matrices $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, as **identity matrices** or **unit matrices** becase they behave like the number 1 in the multiplication of numbers. Note that you must specifically call them **multiplicative identities.** Point out the characteristic that the above identities have 1's in the leading diagonal and 0's elsewhere.

The syllabus is quiet about the zero matrix. However, it is necessary to include it. A matrix with all its elements being zeros is called a zero matrix denoted **O.** Note that a zero matrix can be of any order, not necessarily square. If you add a zero matrix to another matrix, **M** of the same order, what is the result? Your students should be able to tell you that **M** + **O** = **O** + **M** = **M,** provided that **O** and **M** have the same order. Thus, a zero matrix is an **additive identity.** Let students discover that **OM** = **MO** = **O** for any square matrices **O** and **M** of the same order. Note that if M is not a square matrix **OM** = **O** only when **O** and **M** are compatible, e.g.

$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix}$ but $\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

is impossible.

Ask the students to do Exercise 1.6.

Answers to exercises

Exercise 1.1 *(page 2)*

1. (a) $\begin{pmatrix} 7 & 2 & 5 \\ 1 & 2 & 9 \\ 5 & 8 & 1 \\ 0 & 3 & 2 \end{pmatrix}$, 4 × 3

(b) $\begin{pmatrix} 2 \\ 2 \\ 8 \\ 3 \end{pmatrix}$, 4 × 1

(c) (5 8 1), 1 × 3

3. (a) 8

(b) 8

(c) 16

(d) 3

(e) 1

(f) mn

4. (a) 5

(b) 2

(c) 7

(d) 0

5. (a) $a_{2,3}$

(b) $a_{3,1}$

(c) $a_{1,1}$

(d) $a_{2,2}$

6. $\begin{pmatrix} 9 & 13 & 6 & 0 \\ 0 & 7 & 10 & 8 \\ 1 & 18 & 0 & 15 \end{pmatrix}$ when the names of the salesgirls and perfume types are arranged in alphabetical order.

Exercise 1.2 *(page 3)*

1. (a) $\begin{pmatrix} 1 & ^{-}1 \\ 2 & ^{-}1 \end{pmatrix}$

(b) Not possible

(c) $\begin{pmatrix} 3\frac{1}{2} & 3 \\ -\frac{1}{2} & 5\frac{1}{2} \\ 1\frac{1}{2} & 3 \end{pmatrix}$

(d) Not possible

Not possible to add in some cases because of different orders.

2. (a) $\begin{pmatrix} ^{-}2 & ^{-}2 & 3 & 5 \\ 0 & 3 & ^{-}3 & ^{-}1 \end{pmatrix}$

(b) $\begin{pmatrix} ^-1 \\ 5 \\ 2 \end{pmatrix}$

(c) Not possible

(d) $\begin{pmatrix} ^-1 & 7 \\ 6 & 8 \end{pmatrix}$

(e) Not possible

(f) (5 1 7)

3. Associative

4. (a) $\begin{pmatrix} 1 & ^-4 \\ 2 & 4 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & ^-1 \\ ^-5 & ^-1 \\ 2 & ^-4 \end{pmatrix}$

5. $a = 3$, $b = {}^-4$

6. It means that the matrices have different orders and therefore addition or subtraction are not possible

Exercise 1.3 *(page 4–5)*

1. (a) $\begin{pmatrix} 6 & 0 \\ 15 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 4 & 8 \\ ^-4 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & 6 \\ 15 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & 7 \\ 3 & 2 \end{pmatrix}$

2. $k = 3$

3. (a) $\begin{pmatrix} -\frac{5}{4} & 2 \\ 3 & -\frac{1}{2} \end{pmatrix}$

(b) $\begin{pmatrix} ^-2 & 2 \\ -\frac{2}{3} & -\frac{4}{3} \end{pmatrix}$

4. $x = 6, y = 3$

5. $p = 2, q = 2, r = 4$

Exercise 1.4 *(page 6)*

1. (a) (10)

(b) (10)

(c) (55)

(d) not possible

(e) (0)

(f) (0)

2. (a) $x = 2$

(b) $x = 3$

(c) $x = 5.6$

(d) $x = \pm 5$

3. (a) $\begin{pmatrix} 500 \\ 1\ 000 \end{pmatrix}, \begin{pmatrix} 40 \\ 85 \end{pmatrix}$

(b) (24 42)

(c) 45 kg

(d) K 3 720

4. K 5 750

5. K 180

Exercise 1.5 *(page 8–9)*

1. Possible in (a), (c), (d) and (f).

(a) $\begin{pmatrix} 13 \\ 11 \end{pmatrix}$

(c) (16)

(d) (16)

(f) $\begin{pmatrix} 5 & 11 \\ 10 & 22 \end{pmatrix}$

2. AB = $\begin{pmatrix} 10 & 15 \\ 10 & 10 \end{pmatrix}$**, BA =** $\begin{pmatrix} 1 & 3 \\ 23 & 19 \end{pmatrix}$ Yes

3. $k = 4$

4. (a) $x = 2, y = 3$

(b) $x = 3, y = 1$

5. 1×3

6. (a) $\begin{pmatrix} 4 & 3 \\ 5 & 6 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 6 \\ 6 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$

(f) $\begin{pmatrix} pw + qy & px + qz \\ rw + sy & rx + sz \end{pmatrix}$

7. (a) **BC** = $\begin{pmatrix} ^-1 & ^-3 \\ 5 & 11 \end{pmatrix}$,

A(BC) = $\begin{pmatrix} ^-3 & ^-9 \\ 15 & 33 \end{pmatrix}$

(b) **AB** = $\begin{pmatrix} ^-3 & 0 \\ 3 & 6 \end{pmatrix}$, **A(BC)** = $\begin{pmatrix} ^-3 & ^-9 \\ 15 & 33 \end{pmatrix}$

8. (a) $\begin{pmatrix} 4 & 12 \\ 8 & 16 \end{pmatrix}$

(b) $\begin{pmatrix} ^-2 & ^-4 \\ 4 & 10 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 8 \\ 12 & 26 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 8 \\ 12 & 26 \end{pmatrix}$

9. (a) K 60,

(b) (i) $\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$, (ii) $\mathbf{B} = \begin{pmatrix} 360 & 350 \\ 300 & 270 \end{pmatrix}$, (ii) $\mathbf{P} = \begin{pmatrix} 1620 & 1510 \\ 1680 & 1590 \end{pmatrix}$,

How much each lady would spend at each shop,

$\mathbf{BA} = \begin{pmatrix} 1770 & 1780 \\ 1410 & 1440 \end{pmatrix}$, Nothing.

10. Class 4N buys : 56 textbooks, 51 exercise books
Class 4S buys : 54 textbooks, 48 exercise books

Exercise 1.6 *(page 10)*

1. (a) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

2. (a) $\mathbf{I}_2$ as $\mathbf{I}_2\mathbf{M}$ or $\mathbf{MI}_2$

(b) $\mathbf{I}_2$ as $\mathbf{I}_2\mathbf{N}$ or $\mathbf{I}_2$ as $\mathbf{NI}_2$

(c) $\mathbf{I}_2$ as $\mathbf{I}_2\mathbf{P}$ or $\mathbf{I}_2$ as $\mathbf{PI}_2$

3. $\begin{pmatrix} 6 & 1 \\ 2 & 3 \end{pmatrix}$ for both (a) and (b)

4. $\begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix}$, No.

5. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, No, Yes.

6. e.g $a = 2$, $b = 2$, $c = 1$, $d = 1$; etc.

2 TANGENTS TO CIRCLES

(Student's Book pages 11–20)

Background knowledge

Arc of a circle, circumference of a circle, parts of a circle, use of Pythagoras theorem, angle bisector, perpendicular bisector of a line.

Objectives

By the end of this chapter, the student should be able to;

(a) define a tangent to a circle,
(b) deduce from measurement that a tangent is perpendicular to the radius at the point of contact,
(c) show that tangents to a circle from an external point are equal,
(d) illustrate that if two circles touch externally or internally, the point of contact lies on a straight line through the centres,
(e) identify angles between a chord and a tangent,
(f) identify angles in alternate segments,
(g) illustrate the angles in alternate segments are equal,
(h) apply the principles in solving problems,
(i) construct a tangent to a circle,
(j) construct tangents from an external point.

Subtopics

- Tangent to a circle.
- Construction of tangent to a circle:
  - Constructing a tangent at any given point on the circle.
  - Constructing tangents to a circle from a common point.
- Common tangents to two circles:
  - Constructing transverse common tangents to two circles.
- Contact of two circles.
- Angles in alternate segment.

Resources

- Circular shapes,
- Charts showing various types of circles,
- Geometrical instruments.

Teaching guidelines

Tangent to circle

Use appropriate charts to illustrate and define a tangent to a circle. Discuss with students the difference between a tangent and a chord, and the properties of a tangent.

Take students through Example 2.1, then ask them to do Exercise 2.1.

Constructing of tangents to a circle

Guide students through the procedure of constructing a perpendicular through a given point on a circle. Check and ensure that students do accurate construction.

Constructing tangents to a circle from a common point

Guide students through the procedure of constructing tangents to a circle from a common point not on the circle. Ensure students do accurate construction.

Ask students to do Activity 2.1. Discuss their observations and deduce the properties of the tangents constructed. Take students through Example 2.2 then ask them to do Exercise 2.2.

Common tangents to two circles

Use appropriate charts to illustrate direct common tangents and transverse common tangents to two circles as in Fig. 2.11.

Take students through Examples 2.3 and 2.4 before working on Exercise 2.3.

Ask students to do Exercise 2.3.

Contact of circles

Let students draw two circles that touch each other externally. Lead them to deduce the properties of circles in contact.

Do likewise for circles that have internal contact. Lead them through Activity 2.2 and 2.3 to help them understand the properties of circles in contact.

Take students through Example 2.5 and ask them to do Exercise 2.4.

Angles in alternate segment

Using a figure similar to Fig. 2.22, define angles in alternate segment. Ask students to do Activity 2.4. Encourage them to use circles of different radii. Discuss the observations with students and let them come up with the properties of angles in alternate segment. Take them through the analytic proof of the alternate segment theorem, then go through Example 2.6.

Ask students to do Exercise 2.5.

Answers to exercises

Exercise 2.1 *(page 12)*

1. (a) 42.5°

(b) 64°

(c) 108°

(d) 146°

2. (a) 57

(b) 30

(c) 65

(d) 14.2°

6. 16.61 cm

7. 15 cm

Exercise 2.2 *(page 14)*

1. (a) 56°

(b) 29°

(c) 75°

(d) 16°

2. (a) 5.4 cm

(b) 50°

3. (a) 24.6 cm

(b) 26.8 cm

(c) 15.26 cm

4. 13.4 cm

5. 7.334 cm

6. 11.12 cm

7. (a) 4.5 cm

(b) 96°

8. 41°

Exercise 2.3 *(page 15)*

1. (a) 19.60 cm

(b) 12 cm

2. 8.0 cm

3. 6.7 cm

4. (a) 30°

(b) 53.13°

5. (a) $\sqrt{d^2 - (R - r)^2}$

(b) $\sqrt{d^2 - (R + r)^2}$

6. 12 cm

7. 17 cm

Exercise 2.4 *(page 17)*

1. 2.5cm, 1.5cm, 4.5cm

2. 4.5 cm

3. 15.49 cm

4. 3.2 cm, 3.6 cm, 13.4 cm

5. $2\sqrt{Rr}$

7. 130°

8. 2 cm

9. (a) 17 cm

(b) 8 cm

Exercise 2.5 *(page 19–20)*

1. (a) 79°

(b) 62°

2. (a) 66°

(b) 68°

(c) 52°

(d) 110°

3. ∠EDF = 59.5°, ∠DEF = 52.5°, ∠EFD = 68°

4. 57°

6. 78°

8. 30°

9. 144°

10. 36°

3 STATISTICS III

(Student's Book pages 21–29)

Background knowledge

Statistics knowledge learnt in Book 2.

Objectives

By the end of this chapter, the student should be able to:

(a) claculate the mean of data,

(b) calculate variance of data,

(c) calculate the standard deviation of ungrouped data.

Subtopics

- Review of measures of central tendancy.
- Point of interest.
- Measures of dispersion.
- Range
- Mean deviation.
- variance and standard deviation.

Resources

- Square boards
- Graph papers
- Data from real life situations

Teaching guidelines

Review of measures of central tendency

It is a good idea to start by briefly revising the statistics learnt in Book 1. Let Students go through Exercise 3.1 which helps them revise this knowledge.

Alternative method of finding the mean

By using simple distributions as in the Student's Book, let students see the effect, on the mean, of adding or subtracting the same value to/from every data item in a distribution. Define the constant as the assumed or working mean and take the Students through the steps of using an assumed mean to calculate the mean of a distribution, as shown in Example 3.1 in the Student's Book. Take the students through Example 3.2, where they see the application of these steps and the tabular arrangement of the working.

Ask students to do Exercise 3.2.

Measures of dispersion

Explain to the students the concept of dispersion in terms of variation of the data items in a distribution from a central value such as the mean, median or mode. Thus, a measure of dispersion should tell how, on average, the data items are spread out from a chosen average.

Take students through the various measures of dispersion, starting with the easiest to calculate, i.e. the range.

Range

This is the simplest measure of dispersion and your students should not have any problem calculating it. Example 3.3 illustrates the working.

Mean deviation (MD)

Often, Students calculate the mean deviation by simply taking each value, subtracting the mean from it and finding the mean of the deviations. Naturally, they get zero and say that there is no deviation, yet inspection shows that not all the values are equal to the mean. Help them understand that "mean deviation" actually means "mean absolute deviation" as we are not interested in the direction of the variation, but only in the fact that a value is different from the mean (and by how much). Also let Students note that mean deviation may be calculated from any of the averages (only that mean deviation from the mean is the one preferred).

Ask Students to do Exercise 3.3.

Variance and standard deviation

When calculating the mean deviation, we ignore any negative signs on the deviations and take only the magnitude. Instead of this approach, we could ensure that we have only positive values by squaring each deviation. This way, we get the "variance", i.e. the "mean squared deviation".

Take students through the procedure of finding the variance, as in the Student's Book, and come up with the basic formula for the same.

Point out the fact that the variance has different units from the data from which it is calculated. Also, point out, as in the Student's Book, the properties that any measure of spread needs to have for it to be useful. Identify the property that the variance does not meet, but see that we need to take the square root of the variance for the property to be restored. Note that all members of the distribution should be taken into account, but extreme values must not influence the spread unduly. If they are too extreme they may have to be disregarded.

You may now give the name "standard deviation" to the square root of the variance: It is also known as the "root mean squared deviation".

After going through Example 3.4, ask students to work through Exercise 3.4.

(a) Computational method

The formula for this method is found by simply expanding the basic formula, obtained in the previous section. Take students through the expansion as in the Students' Book and through Example 3.5.

(b) Alternative method of calculating standard deviation

The question here is, "What happens to the standard deviation if we add or subtract a constant to/from each of the values of a distribution". Take students through Example 3.6 in order to answer this question. This should enable you to generalise and come up with the formula. Students can now work through Exercise 3.5.

Answers

Exercise 3.1 *(page 21)*

1. (a) Mean = 7.727, Median = 8, Mode = 8

(b) Mean = 13.9, Median = 13.5, Mode = 13

2. (a) 50

(b) 60

(c) 45

3. (a) 40

(b) 35.8

(c) 36

(d) 35

4. (a) Mean = 61.5, Median = 61.9

(b) Modal class is 61–70

Exercise 3.2 *(page 23–24)*

1. (a) 185

(b) 68.94

(c) K 18.70

(d) 224 cm

2. 40.96

3. 59.5 kg

4. 21 years

5. 61.7 cm

Exercise 3.3 *(page 25)*

1. (a) 19, 5.333

(b) 71, 18.6

2. (a) 5

(b) 0.96

3. Mean = 68 kg, MD = 4.533 3 kg.

Exercise 3.4 *(page 26–27)*

1. 10, 2.673

2. 50, 9.899

3. 6.5, 2.327

4. 4, 1.571

5. 37.5, 13.71

6. 27.2, 7.935

Exercise 3.5 *(page 28–29)*

1. Mean = 12.15, s = 1.53

2. s = 15.97

3. Mean = 2.78 cm, s = 0.66 cm

4. Mean = 84.6 marks, s = 6.65 marks

5. Mean = 54.7 kg, s = 9.693 kg

6. Mean = 32.2 marks, s = 7.935

7. Mean = 62.9 kg, s = 7.28 kg

8. (a) 24.53 g

(b) 14.83 g

9. Mean = 47.02 years, s = 12.40 years

4 SIMULTANEOUS LINEAR AND QUADRATIC EQUATIONS

(Student's Book pages 30–35)

Background knowledge

Solving quadratic equations by factor method, completing the square method, quadratic formula.

Solving simultaneous linear equations by substitution and graphical methods, drawing linear and quadratic graphs, reading values from graphs.

Objectives

By the end of this chapter, the students should be able to:

(a) calculate the solutions of simultaneous linear and quadratic equations by substitution.

Subtopics

- Revision.
- The substitution method.

Resources

- Square boards
- Graph papers or books
- Calculators
- Square root tables
- Computer.

Teaching guidelines

Revision

Ensure that the students actually do this work especially the methods of solving quadratic equations. These methods will be applied in the next section of this chapter. Remind them that linear simultaneous equations give a pair of solutions, while those involving quadratic equations must give two pairs of solutions as is illustrated in Example 4.4. Ensure that students go through all the examples and then do Exercise 4.1. Take this opportunity to do thorough revision of this work.

The substitution method

Clearly explain to the students that the purpose of this method is to first combine the given equations into a single quadratic equation in one variable, which can be solved by any of the three methods referred to above as may be appropriate.

Explain also that the linear expression that we use must be in the simplest form possible. Refer the students to the general example used in the student's book.

Take them through the worked examples before they can do the given Exercise 4.2.

Answers

Exercise 4.1 *(page 32)*

1. $x = 4, y = –3$

2. $x = 2, y = –2$

3. $x = 2, y = 3$

4. (a) When $x = –1, y = 4$
When $x = 4, y = 6$

(b) $x = –2$ or $–3$

5. When $x = 2.25, y = 5.5$
When $x = –2.75, y = 4.5$

Exercise 4.2 *(page 35)*

1. When $x = 1.5$, $y = 0$; When $x = 0$, $y = 8$

2. When $x = 2$, $y = 4$; When $x = 1.5$, $y = 3.75$

3. When $x = -1.6$, $y = 0.61$
When $x = 2.1$, $y = 6.2$

4. $x = \pm 2$, $y = \pm 2$

5. When $x = 2$, $y = 1$; When $x = 1$, $y = 4$

6. When $x = 0.9$, $y = 0.2$; When $x = -1.9$, $y = 3$

7. When $x = 0.4142$, $y = 0.7574$
When $x = 2.4142$, $y = -5.2426$

8. When $x = 0$, $y = 2$; When $x = 2.5$, $y = 12$

9. When $x = 1.3$, $y = 8.5$; When $x = 0.2$, $y = 3$

10. When $x = -0.9$, $y = -2.8$
When $x = 3.4$, $y = 5.8$

11. When $x = 3$, $y = 1$; When $x = -1$, $y = -3$

12. When $x = 6$, $y = 3$; When $x = -4$, $y = -27$

13. When $x = 4$, y = 5; When $x = -1$, $y = -5$

5 PROGRESSIONS

(Student's Book pages 36–40)

Background knowledge

Shapes, patterns, natural numbers, multiplication, division, addition, ratio.

Objectives

By the end of this chapter, the student should be able to:

(a) recognise an AP,
(b) calculate common difference of an AP,
(c) calculate the n^{th} term of an AP,
(d) recognise the general form of an AP,
(e) use the formula for the n^{th} term to calculate the common difference and the number of terms,
(f) calculate the sum of terms of an AP using the formulae,
(g) solve problems involving APs,
(h) calculate common ratio of a GP,
(i) calculate the n^{th} term of a GP,
(j) recognise the general form of a GP,
(k) use the formula for the nth term to calculate ratio and the number of terms,
(l) use the formula to calculate the sum of GPs,
(m) solve real life problems involving GPs.

Subtopics

- Series
  - Arithmetic progression (A.P)
    [Arithmetic series].
  - Geometric progression (G.P)
    [Geometric series].
  - Application of A.P and G.P to real life situations.

Resources

- Charts illustrating number patterns
- Calculators
- Bottle tops
- Match sticks
- Match boxes

Teaching guidelines

As a revision exercise of the work on sequences already done and in preparation for the work on arithmetic and geometric progressions use the suggested resources i.e. match sticks to demonstrate how to build simple arithmetic and geometric sequences. For example, a simple sequence like 1, 2, 3, 4, …. can be demonstrated using match sticks as I, II, III, IIII, ….. In this demonstration one stick stands for 1, two sticks for 2 and so on. Similarly, a geometric sequence such as 2, 4, 8, 16…. can also be demonstrated using items like bottle tops as;

○ ○○ ○○○○ ○○○○○○○○

○, ○○, ○○○○, ○○○○○○○○,… and so on.

You must appreciate that these demonstrations are only meant to help the students to visualise the pattern and not practical for large numbers. You can now proceed to the work on progressions.

Series

Use sequences already met to define a series. Let students identify finite and infinite series.

Arithmetic progression (A.P) [Arithmetic Series]

Use arithmetic sequence already met to define Arithmetic progression (A.P) [or series]. Let students give examples of arithmetic progressions.

Guide students in stating the general form of an arithmetic progression and in the derivation of the formula for partial sum of an arithmetic progression.

Take students through Example 5.1 and ask them to do Exercises 5.1.

Geometric progression (G.P) [Geometric Series]

Guide students in defining a geometric progression. Let them give examples of geometric progression and the general form of a geometric progression.

Guide students in deriving the formula for partial sum of a geometric progression.

Take them through Example 5.2, then ask them to do Exercise 5.2.

Application of A.P and G.P to real life situations

Discuss with students examples of A.P and G.P from real life situation. Take them through Examples 5.3 and 5.4, then ask them to do Exercise 5.3.

Answers to exercises

Exercise 5.1 *(page 37)*

1. (b) 2

(d) 1

(e) 3

(g) 6

2. (a) 91

(b) 735, $\frac{n}{2}(8 + 6n)$

3. (a) 20

(b) 940

4. $S_n = \frac{n}{2}(17 + 3n)$

5. 565

6. 8

7. 13

8. 5 586

9. 8^{th} term = $-2\frac{1}{2}$

Exercise 5.2 *(page 38–39)*

1. (a) 162, 242 $\frac{2}{3}$

(b) $\frac{1}{243}$, 13.5

(c) –256, $\frac{-1\,023}{6}$

(d) $\frac{128}{2\ 187}$ or ≈ 0.06, $\frac{18\ 915}{6\ 561}$ or ≈ 2.88

(e) 0.656 1, 4.095 1

(f) 0.35, 5.54

(g) 1, 50.33

2. (a) 683

(b) 90.91

(c) 99.22

(d) 16.67

(e) –1 302

(f) $\frac{3\ 280}{243}$ or (13.50)

3. 2, –8, 32, –128 or 2, 8, 32, 128

4. –3

5. 10.7

6. 2

7. 9

Exercise 5.3 *(page 40)*

1. 2 007

2. 22 months

3. K 1 600

4. K 6 457

5. 8.75%

6. (a) 1.05p where p = K 2 000

(b) $p\{1.05 + (1.05)^2\}$

(c) $p\{1.05 + (1.05)^2 + (1.05)^3\}$; K 26 413.50

7. 5 million

8. 15 years

1–5 ANSWERS TO REVISION EXERCISES 1

(Student's Book pages 41–44)

Background knowledge

All that has been learnt in chapters 1 to 5.

Specific objectives

By the end of these revision exercises, the student should be able to answer similar exercise/questions accurately and with appropriate speed.

Resources

If any remedial work is needed, use same resources as suggested in the various chapters.

Teaching guidelines

Each revision exercise covers all the five chapters so far done. You may either give them as classroom exercises, with students discussing them in groups, or may use them as quick tests. Find ways of having students develop speed in answering the questions. For example, you may organise speed contests groups. However, remember to emphasise accuracy in the working and the showing of all the steps involved. Whichever approach you use, be on the look out for obvious learning difficulties experienced either by the whole class or by individual students and address them immediately.

Revision Exercise 1.1 *(page 41–42)*

1. $\begin{pmatrix} 13 & 23 \\ 8 & 7 \end{pmatrix}$

2. $\begin{pmatrix} 11 & 8 \\ 17 & -24 \end{pmatrix}$

3. (a) (i) 48°

(ii) 222°

(b) Two angles are equal, hence Δ ABD is isosceles.

4. 4.5 cm

5. Mean = 95, SD = 4.90

6. (a) 16.86

(b) (i) 17.24

(ii) 4.152

7. 10

8. 6 138

9. (a) 131.5

(b) 20

10. (a) $x = 1, y = 6$

(b) $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$

11. (a) 6.28, 0.72

(b) 2.12, –0.79

12. 15 cm, 20 cm

Revision Exercise 1.2 *(page 42–43)*

2. $\underset{\sim}{C} = \begin{pmatrix} \frac{1}{2} & \frac{-1}{4} \\ 1 & \frac{1}{2} \end{pmatrix}$

3. CD = 9.92 cm

4. (a) r = 3.6 cm, R = 5.4 cm

(b) AT = 6.0 cm, TQ = 4.3 cm

5. (a) 15; 3.162

(b) 150; 31.62

(c) 170; 31.62

6. (a) 18

(b) 4.73

(c) 5.49

7. (a) –910

(b) 1.998

8. 5.52 a.m.

9. 169.9

10. (a)

	sugar	beef
Ekari	2	5
Tadala	3	2

(b) Ekari K 2 800
Tadala K 1 450

11. (a) 0.27, –3.77

(b) 3 twice

(c) no real roots

12. (a) $\frac{25}{4}$, $\left(y + \frac{5}{2}\right)^2$

(b) $\frac{169}{4}$, $\left(x + \frac{13}{2}\right)^2$

Revision Exercise 1.3 *(page 43–44)*

1. $\begin{pmatrix} -5 & 9 \\ 7 & 5 \end{pmatrix}$

2. $\begin{pmatrix} 8 & 6 \\ 12 & 14 \end{pmatrix}$

3. (a) $\sqrt{Rr}$

(b) $\frac{Rd}{R-r}$

4. (a) 2.4 cm

(b) 132.4°

(c) 28.72 cm^2

5. Mean = 74.9
Mean deviation = 1.85

6. (a) 2.276

(b) (i) 13.2

(ii) 2.276

7. 45^{th}

8. (a) $a = {}^{-}1$; $d = 2$ (b) 35

9. (a) 45.6

(b) 45.8

11. (a) –2,

(b) $1\frac{1}{2}$,2

(c) –1, 3

(d) $2\frac{1}{2}$, –3

12. (a) $x = 2$, $y = -2$ and $x = -2$, $y = 2$

(b) $x = 4$, $y = 3$ and $x = 3$, $y = 4$

6 TRAVEL GRAPH

(Student's Book pages 45–50)

Background knowledge

Distance and speed as used in everyday life. Plotting and drawing graphs find gradient of a line

Objectives

By the end of this chapter, the student should be able to:

(a) draw speed time – graphs.

(b) find acceleration by calculating the gradient of a line,

(c) find deceleration by calculating the gradient of a line,

(d) calculate the area under speed time graph,

(e) interpret that the area under speed-time graph is equal to the distance covered,

(f) calculate the distance travelled by using '*average speed* x *time*'.

Subtopics

- Speed- time graphs.
- Describing speed – time graphs.
- Average speed, time and distance.

Resources

- Drawn graphs
- Graph boards
- Pencil
- Ruler
- Eraser
- Graph papers.

Teaching guidelines

Speed – time graphs

Guide students through the drawing of speed – time graphs ensuring that they differentiate them from distance – time graphs. Show them how to use speed – time graphs to get acceleration, as the gradient of the graph. Again pay attention to the case of zero gradient and ensure that students understood its meaning.

Emphasize that in speed – time graph, time is displayed on the horizontal axis, while the speed on the vertical axis. Relate the ares bounded by the graph and the time axis to the distance travelled during the time in question.

Take learners through Example 6.1, distinguish between acceleration and deceleration and the appropriate units. Let them do Exercise 6.1, emphasizing and ensuring that they all do the work. In this exercise, illustrate the graphs on the board for the class to see the correct graphs.

Describing speed – time graphs

This section is mainly concerned with consolidating what was learnt in the previous two sections. The emphasis should be on graphs of linear motion that have multiple parts. Ensure that the students see that different parts of the graphs give different information about the motion e.g. different speeds on different acceleration. Note that the graph can also be a curve. In this case, we talk of two types of speed and acceleration, viz. average speed and acceleration, and speed and acceleration at an instant.

Take students through the detailed graph description, using as an example before asking students to do Exercise 6.2.

Average speed, time and distance

Ensure that students are familiar with the relationship between speed, time and distance and that they can use it to find any of the three qualities given adequate information. For the purposes of finding average speed, it is important to emphasize that total time includes rest time in between the journey.

Guide them through Example 6.2 and ensure that they do Exercise 6.3.

Answers to exercises *(page 46–47)*

Exercise 6.1

1. (b) (i) 5 m/s

(ii) 0.25 m/s^2

(iii) 400 m

2. (b) (i) 30 m/s

(ii) 1 m/s^2, 4 m/s^2, 0.5 m/s^2

(iii) 5 m/s, 15 m/s, 25 m/s

(c) 875 m

3. (b) (i) 2.5 m/s^2

(ii) 1.67 m/s^2

(c) 680 m, 55.64 km/h

4. (b) 6 m/s, 20 m/s, 42 m/s

(c) speed varies with time and so we cannot find average at an instant because the graph is a curve.

(d) 4.5 seconds, 7.5 seconds

Exercise 6.2 *(page 48–49)*

1. (a) (i) Increases speed from 20 m/s to 40 m/s in 5 s.

(ii) Increases speed from 0 m/s to 10 m/s in 3 s, increase speed from 10 m/s to 50 m/s in 2 s.

(iii) Moves with constant speed of 20 m/s for 5 s.

(iv) Changes speed from 50 m/s to 0 m/s in 4 s.

(v) Reduces speed from 30 m/s to 0 m/s in 2 s, then increases speed from 0 m/s to 50 m/s in 3 s.

(b) (I) (i) 20 m/s^2

(ii) $\frac{10}{3}$ m/s^2, 20 m/s^2

(iii) 0 m/s^2

(iv) $\frac{-25}{2}$ m/s^2

(v) –15 m/s^2; $\frac{50}{3}$ m/s^2

(II) (i) 150 m

(ii) 75 m

(iii) 100 m

(iv) 100 m

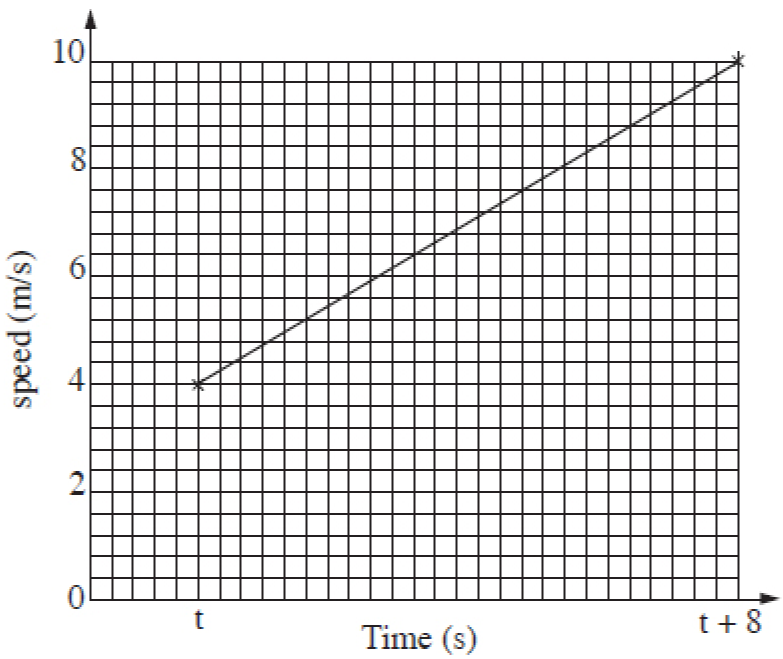
(v) 105 m

2. (a) Car starts with an initial speed of 6 m/s, then accelerates at 1.5 m/s^2 for 8 seconds, continues at that speed for a further 25 seconds.

(b) 28 seconds,

(c) 660 m

3. Acceleration = 0.75m/s^2



4. (a) (i) 0.43 m/s^2

(ii) 0.91 m/s^2

(iii) –2 m/s^2

(b) (i) 94.25 m

(ii) 440 m

5.

Initial speed (m/s)	Final speed (m/s)	Time taken (s)	Acceleration (m/s^2)
10	30	4	5
0	40	5	8
40	0	80	–0.5
2	9.5	5	1.5

6. (a) 1 m/s^2

(b) 275 m

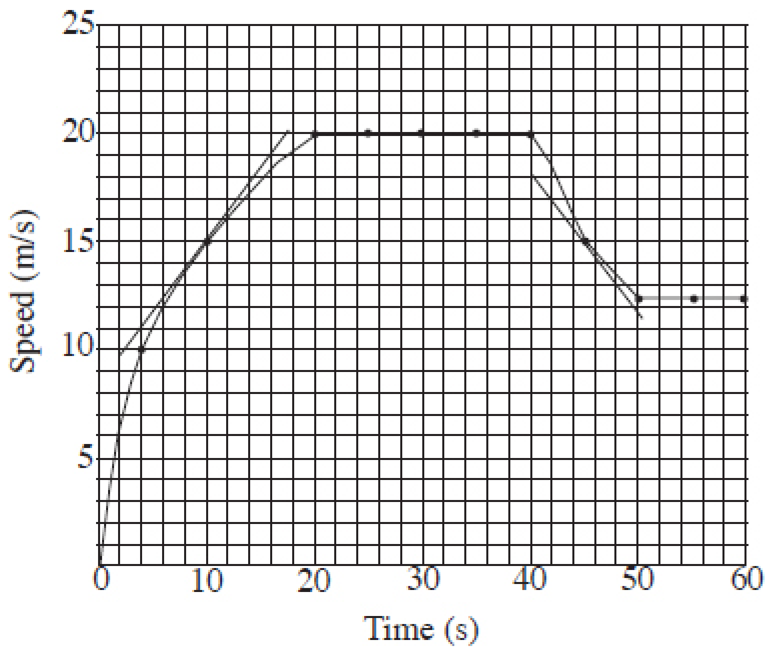
(c) $7\frac{6}{7}$ m/s = 7.857 m/s

7. (a) $1\frac{1}{3}$ m/s^2

(b) 0.8 m/s^2

(c) $^{-}16$ m

8.



At t = 15, speed ≈ 0.73 m/s
At t = 45, speed ≈ 0.82 m/s

Exercise 6.3 *(page 50)*

1. (a) 162 km

(b) 44.18 km/h

2. Distance is 2.7 km

3. Distance between A and B is 360 km

4. Distance = 105 km

5. (a) 72.5 km/h

(b) 2475 km

(c) $2\frac{1}{2}$ hrs

(d) (i) 525 km

(ii) 70 km/h

7 TRIGONOMETRY II

(Student's Book pages 51–59)

Background knowledge

Knowledge of drawing a circle, angles at a point, degree measure of angles, definition of sine, cosine and tangent of an angle θ, coordinates of points on Cartesian graph.

Objectives

By the end of this chapter, the student should be able to:

a) calculate area of a triangle using area rule,
b) calculate the angles of a triangle using area rule,
c) calculate a side of a triangle using sine rule,
d) calculate an angle using a sine rule,
e) calculate a side of a triangle using cosine rule,
f) calculate an angle of triangle using cosine rule,
g) solve problems using sine/cosine rules,
h) sketch bearing of a point,
i) calculate bearing of a point using sine/cosine rule.

Subtopics

- Introduction
- Area rule
- Sine rule
- Cosine rule
- Bearings.

Resources

- Chart illustrating a unit circle
- Charts illustrating graphs of sin θ, cos θ, tan θ
- Mathematical tables
- Calculators

Introduction

Discuss with students the definitions of trigonometric ratios: sine, cosine and tangent of angles as a review of what they learned in Book 2.

Teaching guidelines

The Area rule

Students are already familiar with the method of finding (a) the area of a right angled triangle and (b) area of any triangle given the base and the corresponding altitude. Remind students that any of the three sides of a triangle can be used as the base with the appropriate height. You may illustrate this using triangle ABC as in Figure 7.1.

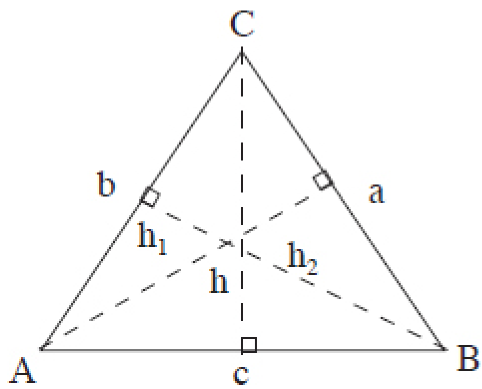


Fig. 7.1

Area of ΔABC can be expressed in three different ways i.e.

(i) Using base AB, and altitude from C; Area = $\frac{1}{2}$ ch.

(ii) Using base BC and altitude from A; Area = $\frac{1}{2}$ ah_1

(iii) Using base AC and altitude from b; Area = $\frac{1}{2}$ bh_2

Thus, $\frac{1}{2}ch = \frac{1}{2}ah_1 = \frac{1}{2}bh_2$

It is on this foundation that the sine formula is built. Take students through Example 7.1.

After deriving the sine formula for the area of ΔABC i.e. Area = $\frac{1}{2}$ ab sin C, let the students find equivalent formulas for the same area in terms of sin A and sin B for both acute and obtuse angled triangles.

Guide students through Examples 7.2 and 7.3 and ask them to do Exercise 7.1 Questions 1, 2, 3.

The sine rule

Discuss with students how to derive the sine rule. It would be useful if students did the work step by step in their exercise books as you work on the chalkboard so that they are actively involved in the derivation. Ensure that diagrams used are large enough and are clearly labelled. Discuss the use of sine rule in solving triangles as in Examples 7.4 and 7.5, ensuring that students are able to identify situations where they can apply the sine rule.

Ask the students to do Exercise 7.2.

Cosine rule

As in the case of the sine rule, let students be involved in the derivation of the cosine rule at each step.

Discuss the use of the cosine rule in solving triangles. Again ensure that students are able to recognise situations where they can apply the rule and where they cannot apply it.

Take them through Examples 7.6 and 7.7 and if need be, give them more examples.

Ask the students to do Exercise 7.3.

Bearing

In Book 3, we used the method of scale drawing to locate given points and to find bearings and distances between such points. In this section, we are going to apply the sine and cosine rules to find distances and bearings. Example 7.8 illustrates this application. Discuss Example 7.8 and any such examples, with students and its solution step by step.

Ask students to do Exercise 7.4.

Answers to exercises

Exercise 7.1 *(page 53)*

1. (a) 48.30 cm^2

(b) 15.35 cm^2

(c) 17.61 cm^2

(d) 9.193 cm^2

(e) 18.07 cm^2

(f) 12.94 cm^2

2. (a) 35.98°

(b) 68.01°

(c) 90°

(d) 90°

(e) 34.91°

(f) 125.1°

(g) 140°

3. (a) 7.632 cm^2

(b) 9.193 cm^2

(c) 9.642 cm^2

(d) 6 cm^2

(e) 5.283 cm^2

(f) 8.863 cm^2

(g) 11.57 cm^2

(h) 21.22 cm^2

Exercise 7.2 *(page 55)*

1. (a) $\angle$A = 18.86°, $\angle$B = 125.14°, b = 5.56 m

(b) $\angle$B = 110°, a = 3.76 cm, c = 2.25 cm

(c) $\angle$C = 36°, b = 5.17 cm, c = 7.47 cm

(d) $\angle$A = 96.8°, $\angle$C = 55.2°, a = 8.46 cm

(e) $\angle$B = 50°, $\angle$C = 80°, c = 7.71 cm

(f) $\angle$C = 70°, b = 2.54 cm, c = 4.77 cm

2. $\angle$Q = 14.5°, r =14.20 cm

3. x = 16.3 cm, y = 18.4 cm

4. (a) q = 2.66 cm, $\angle$R = 80°, r = 5.24 cm

(b) ∠R = 39°, p = 6.85 cm, *r* = 4.46 cm

5. 8.16 cm

6. 22.6 cm

7. 28.4 cm

8. 177 million km

9. 4.25 m

10. PQ = 28.8 cm, QR = 26.0cm

Exercise 7.3 *(page 57–58)*

1. (a) ∠A = 38.2°, ∠B = 81.8°, ∠C = 60°

(b) ∠B = 22°, ∠C = 63.0°, *a* = 16.2 cm

(c) *b* = 19.0 cm, ∠A = 61.8°, ∠C = 38.2°

(d) ∠A = 25.9°, ∠B = 115.2°, ∠C = 38.9°

(e) ∠A = 65°, ∠B = 65°, *c* = 5.07 cm

(f) ∠A = 26.7°, ∠B = 36.5°, ∠C = 116.8°

2. 10.87 cm

3. 3.40 cm

4. 54.7°

5. ∠A = 56.4°, ∠C = 93.6°, *c* = 5.99 cm

6. 3.8 cm

7. 6.46 cm, 7.89 cm

8. 12.7 cm

9. 3.97 cm, 7.50 cm

Exercise 7.4 *(page 59)*

1. 95.75 km

2. 95.82 million kilometres

3. 260.92 km

4. 64.3°

5. 204.2 km

6. CA = 9.3 cm, CB = 10.8 cm
Hence A is nearer

7. 726.14 km

8. 10.3 m

9. (a) 484.70 m,

(b) 55.4°

8

POLYNOMIALS

(Student's Book pages 60–66)

Background knowledge

Factorisation of binomials, Expansion of binomials, Substitution, Solution of simple equations.

Objectives

By the end of this chapter, the student should be able to:

(a) state degree of a given polynomial,
(b) divide a polynomial of higher degree by a polynomial of a lower degree,
(c) find the remainder using remainder theorem,
(d) factorise polynomials of third degree,
(e) find roots of polynomial equations of third degree,
(f) find polynomial coefficients in identical polynomials.

Subtopics

- Definition of a polynomial.
- Addition and subtraction of polynomials.
- Multiplication and division of polynomials.
- The remainder theorem.
- The factor theorem.
- Cubic equations.
- Identities.

Resources

- Charts

Teaching guidelines

Addition and subtraction

Give a clear definition of a polynomial distinguishing between the coefficient, the variable and the degree. A polynomial must be in one variable.

Emphasize on the importance of the ability to identify like terms for the purposes of addition and subtraction. Give the students an opportunity to expand simple binomial expressions to ensure that they are ready for polymonial multiplication. Lead them through example 8.1.

Division

Long division of numbers is a necessary skill in the division of polymonials. Emphasize on the importance of order in both the divided and the divisor. Any missing term in both the divisor and the dividend *must* be replaced with a zero term, otherwise the division cannot be done. Define the quotient and the remainder and ensure that the students are able to express a polynomial in terms of the quotient, the divisor and the remainder.

Take students through examples 8.3 to 8.5 before they can do Exercise 8.1.

Explain the meaning of the word variety as used in Question 4 of Exercise 8.1. It would be beneficial to your students if you illustrate this verification on the chalkboard.

The remainder theorem

Take students through the process of looking for the remainder as explained in the Student's Book. Ensure that they can state the theorem correctly and test for the remainder in a polymonial division without doing the actual division.

The factor theorem

State the factor theorem and distinguish it from the remainder theorem.

Explain how we use the factor theorem to find factors of a polynomial. This is very well explained in Examples 8.7 and 8.8. Now let the students do Exercise 8.2.

Cubic Equations

Define a cubic equation and describe possible methods of solving a cubic equation.

Explain how we use the constant term of the polynomial to find its factors where they exist. Example 8.9 illustrates this process very well. Students now do Exercise 8.3

Identities

Define an identity by stating its properties as described in the Student's Book. Give simple examples of identities take time to take students through Examples 8.10 and 8.11 and if need be formulate some more examples. Ensure that students work through Exercise 8.4.

Answers to exercises

Exercise 8.1 *(page 62)*

1. (a) $-x^4 - 12x^3 + 3x^2 + 7x + 10$

(b) $-3x^5 + 4x^3 + 6x^2$

(c) $12a^3 + a^2 + 4a - 12$

(d) $4a^5 + a^4 + 3a^3 - 7a^2 - 5a$

2. (a) $4x^3 + 0x^2 - 2x + 8$

(b) $4x^5 + 0x^4 - 3x^3 - 2x^2 + 7x + 0$

(c) $8x^5 + 0x^4 + 4x^3 - 3x^2 + 0x - 7$

(d) $3a^4 + 0a^3 - 8x^2 + 7a - 0$

3. (a) 4m

(b) $16x$

(c) $5x$

(d) $10x$

4.

5. (a) Q = $3x + 7$; R = 0

(b) Q = 4t – 3; R = 8

(c) Q = –2a^3 – 3; R = – 1

(d) Q = –4r; R = 9

6. (a) Q = $-x^2 - x + 1$; R = 1

(b) Q = $2a^2 - 24$; R = $a + 5$

(c) Q = y – ; R = – 1

(d) Q = $3a^2 + 3a + 3$; R = 2

(e) Q = $h^2 - 4h - 2$; R = 0

Exercise 8.2 *(page 64)*

1. (a) 0

(b) 12

(c) 2

(d) –14

2. (a) (3x – 1) and $(x + 1)$

(b) $(x + 1)$ and $(x - 4)$

(c) (x – 1) and $(x + 3)$

(d) $(x - 1)$ and $(x + 2)$

3. (a) $-15x^3 - 28x^2 + 5x + 2 \equiv (x + 2)(1 - 3x)(1 + 5x)$

(b) $x^3 - 4x^2 + x + 6 \equiv (x + 1)(x - 2)(x - 3)$

(c) $x^3 - 8x^2 + 19x - 12 \equiv (\mathrm{x} - 2)(\mathrm{x} - 6)(\mathrm{x} - 1)$

(d) $x^3 - 2x^2 - 5x + 6 \equiv (x - 1)(x + 2)(x - 3)$

(e) $x^3 + 1 \equiv (x + 1)(x^2 - x + 1)$

(f) $x^3 - 1 \equiv (x - 1)(x^2 + x - 1)$

4. (a) R = 5

(b) R = 0

5. (a) $3x^3 - x^2 - 6x + 4 \equiv (x - 2)(3x + 2)(x - 1)$

(b) $2x^3 - 4x^2 - 9x + 9 \equiv (x - 3)(2x - 1)(x + 3)$

(c) $4x^s - 5x^2 - 18x - 9 \equiv (x - 3)(4x + 3)(x + 1)$

(d) $x^3 + 4x^2 - 4x - 16 \equiv (x - 2)(x + 4)(x + 2)$

Exercise 8.3 *(page 65)*

1. (a) $x = \{1, -2, 3\}$

(b) $x = \{-1, 3, 8\}$

(c) $x = \{-4, 1, -1/5\}$

2. $\{3, -2, -1\}$

3. $\{-1, 3, \frac{1}{2}\}$

4. $\{3, -\frac{1}{2}, -9\}$

5. $\{-2, -\frac{1}{3}, 5/2\}$

6. $\{-3, \frac{1}{2}, 2\}$

7. $\{2, -3, 4\}$

8. $\{2, -\frac{1}{2}, -2\}$

9. $\{\pm 1, \frac{2}{3}\}$

10. {2, 1 twice}

Exercise 8.4 *(page 66)*

1. $K = -8$

2. $x^3 + 3x^2 - 4x - 12 \equiv (x - 2)(x + 2)(x + 3)$

3. $a = -1$
$b = 8$

4. $K = -7$

5. $a = -7$
$b = 5$
$R = 73$

6. $a = -3$
$b = -32$

9 PROBABILITY II

(Student's Book pages 67–76)

Background knowledge

Fractions and decimals, length, area, volume.

Objectives

By the end of this chapter, the student should be able to:

(a) define probability space,
(b) construct a probability space table,
(c) calculate probability problems using probability space,
(d) determine experimental probability of events,
(e) construct a tree diagram,
(f) calculate probability of an event using tree diagram.

Subtopics

- Sample space.
- Experimental probability.
- Probability space.
- Probability involving two events.
- Tree diagram.

Resources

- Coins.
- Dice.
- Deck of playing cards.
- Probability games.
- Calculators.

Teaching guidelines

The possibility space or sample space

Using various examples, discuss and define a possibility space. Help students to go through Example 9.1 and ask them to do Exercise 9.1.

Experimental probability

Using appropriate examples, help students to understand what is meant by such terms as random selection, at least, at most, not more than, not less than. Also explain what is meant by the term "odds".

Ask students to do Exercise 9.2.

Probability space

Discuss and define discrete probability space. See that students are able to list down probabilities in a probability space.

Go through Examples 9.5 and 9.6 with students.

Using results of questions 5, 6 and 7 of Exercise 9.1 and other such examples, define continuous possibility space. Discuss with students Example 9.2 and any other similar examples.

Ask students to do Exercise 9.3.

Probability involving two events

Discuss with students examples of mutually exclusive events. Use Example 9.7 to deduce the addition rule for probabilities of mutually exclusive events.

Ask students to do Exercise 9.4.

Ensure that you discuss with the students, the results to Question 11 of Exercise 9.4.

Tree diagram

Discuss, with students, solution of probability problems and lead them to appreciate the use of a tree diagram in solving problem. Some problems can be quite cumbersome, but use of a tree diagram makes the working easier.

Ask students to do Exercise 9.5.

Ensure that you discuss the student's results to Question 11 of Exercise 9.4.

Answers to exercises

Exercise 9.1 *(page 68–69)*

1. H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6; 12 outcomes

2. (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) – 36 outcomes

3. 1, 2, 3, … – Infinite

4. HH, HT, TH, TT – Four

5. $x \geq 0$ where x is distance between coins – Infinite

6. A range of values from lowest height to the highest – Infinite

7. An interval of time, but limits not possible to set – Infinite

8. $\frac{1}{4}$

9. $\frac{5}{9}$

10. 0.419

11. 0.090 0

12. 0.463 6

13. 0.044 89

14. $\frac{19}{60}$

15. (a) $\frac{1}{20}$ (b) $\frac{1}{10}$ (c) $\frac{5}{192}$ (d) $\frac{31}{576}$ (e) $\frac{1}{768}$
(f) 0.07474

Exercise 9.2 *(page 70–71)*

1. (a) $\frac{1}{6}$ (b) $\frac{2}{3}$

2. (a) $\frac{5}{12}$ (b) $\frac{1}{4}$ (c) $\frac{5}{18}$ (d) $\frac{1}{6}$

3. (a) $\frac{1}{9}$ (b) $\frac{1}{6}$ (c) $\frac{5}{12}$

4. $\frac{3}{5}$

5. (a) $\frac{5}{8}$ (b) $\frac{7}{8}$ (c) $\frac{3}{4}$

6. $\frac{6}{25}$

7. $\frac{5}{8}$

8. $\frac{3}{4}$

9. (a) $\frac{1}{5}$ (b) $\frac{9}{25}$ (c) $\frac{7}{25}$

10. $\frac{5}{28}$

11. $\frac{1}{7}$

12. (a) $\frac{5}{8}$

(b) K 187.50

13. (a) $\frac{4}{20}$ (b) $\frac{3}{20}$ (c) $\frac{11}{20}$

14. (a) $\frac{3}{4}$ (b) $\frac{6}{13}$

Exercise 9.3 *(page 72)*

1. (a) Not a probability space because one of the probabilities is negative.

(b) Not a probability space because the sum is greater than 1.

(c) Not a probability space because the sum is less than 1.

2. (a) $\frac{3}{4}$

(b) $\frac{1}{4}$

3. (a) $\frac{43}{72}$ (b) $\frac{33}{80}, \frac{11}{80}$

4. $\frac{3}{20}$

Exercise 9.4 *(page 73–74)*

1. (a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{20}$ (d) $\frac{7}{20}$
   (e) $\frac{9}{20}$ (f) $\frac{3}{4}$

2. (a) $\frac{5}{21}$ (b) $\frac{13}{21}$

3. $\frac{4}{15}$

4. (a) $\frac{1}{2}$ (b) $\frac{2}{13}$

5. $\frac{231}{384}$

6. (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$, sum is 1

7. (a) $\frac{1}{50}$ (b) $\frac{1}{10}$

8. $\frac{3}{5}$

9. (a) $\frac{7}{30}$ (b) $\frac{8}{15}$

10. (a) $\frac{1}{24}$ (b) $\frac{1}{4}$

12. (a) $\frac{204}{1\,015}$ (b) $\frac{297}{1\,015}$

13. $\frac{21}{40}$

14. $\frac{2}{3}$

15. $\frac{1}{120}$

16. (a) $\frac{2}{5}$ (b) $\frac{4}{15}$

17. $\frac{3}{200}$

Exercise 9.5 *(page 76)*

1. (a) $\frac{91}{460}$ (b) $\frac{21}{46}$ (c) $\frac{1}{4}$

2. $\frac{7}{40}$

3. (a) $\frac{13}{20}$ (b) $\frac{19}{40}$

4. $\frac{31}{72}$

5. (a) 0.432

(b) 0.243

(c) 0.108

6. $\frac{10}{49}$

7. (a) $\frac{8}{21}$ (b) $\frac{1}{7}$

8. (a) $\frac{3}{10}$ (b) $\frac{1}{15}$ (c) $\frac{7}{15}$

9. (a) $\frac{10}{63}$ (b) $\frac{5}{42}$

10. (a) $\frac{54}{125}$ (b) $\frac{18}{25}$

10 VECTORS II

(Student's Book pages 77–84)

Background knowledge

Basic operations on numbers, locating and plotting points on the Cartesian plane, use of Pythagoras' theorem.

Objectives

By the end of this chapter, the student should be able to:

(a) present magnitude in different notations,
(b) calculate the magnitude of a vector,
(c) describe zero/null vector,
(d) describe position vector,
(e) find a position vector,
(f) identify parallel vectors,
(g) find mid point of a vector,
(h) show that points are collinear using vector method,
(i) add vectors using parallelogram law,
(j) solve problems by applying a parallelogram law.

Subtopics

- Position vector
- Midpoints
- Magnitude of a vector
- Zero/Null vector
- Parallegram law of addition
- Parallel vectors and collinear points
- Application of parallelogram law.

Resources

- Chalkboard with square grid.
- Graph papers.
- Geometrical instruments.

Teaching guidelines

Position vector

Remind students that given any vector **PQ,** P is the initial point and Q is the terminal point, and the direction is from P to Q.

Using Fig. 10.1, ask students to state the initial and final points of vectors OA, OB, OC and OD. Also ask them to state what is common with all these vectors. Using their responses, i.e. all these vectors start from (0, 0), define **position vector** of a point P as the vector **OP** where O, the origin (0, 0) is the initial point of the vector.

Ask students to state how many units one moves along the x-axis and along the y-axis from the origin O to the points P(1, 2), Q($^-$3, 4), R(4, $^-$2 and S ($^-$4, 1)? So what are the column vectors for position vectors **OP, OQ, OR** and **OS?**

Help them to deduce the relationship between coordinates of a point and its position vector. The students should be able to state the position vector of a point P(x, y) as OP = $\begin{pmatrix} x \\ y \end{pmatrix}$.

Note that we usually denote position vector of A as **a,** of B as **b** and of C as **c** etc.

Discuss Example 10.1 with the students. Discuss how to get, for example, vector AB given coordinates of A and B or their position vectors. Through this example or any other such example, establish that if **OP** = $\begin{pmatrix} a \\ b \end{pmatrix}$

, **OQ** = $\begin{pmatrix} c \\ d \end{pmatrix}$, then PQ = OQ – OP=$\begin{pmatrix} c \\ d \end{pmatrix} - \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} c - a \\ d - b \end{pmatrix}$

Let students now do Exercise 10.1.

Illustrate Q2 of this exercise so that the students may compare their work with your.

Illustrate Question 2 of this Exercise so that the students may compare their work with yours.

Mid-points

In this section, we want to make students learn what is referred to as the mid-point theorem. The theorem states that, given any two points A and B with position vectors **a** and **b** respectively, then the position vector of M, the mid-point of

AB is $\frac{1}{2}$**(a + b)** = $\frac{1}{2}$ **a** + $\frac{1}{2}$ **b.**

However at this stage you should not use the name of the theorem. Using Fig 10.3 in Student's Book, go through the steps shown to establish that position vector.

OM = $\frac{1}{2}$ **a** + $\frac{1}{2}$ **b** = $\frac{1}{2}$ **(a + b)** where **a** and **b** are position vectors of A and B respectively and M is the mid-point of AB.

Remind the students that while dealing with they established;

that if **u** = $\begin{pmatrix} a \\ b \end{pmatrix}$ and **v** = $\begin{pmatrix} c \\ d \end{pmatrix}$,

then **u** + **v** = $\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} c + a \\ d + b \end{pmatrix}$

Ask them to use this fact to establish that given points A(a, b) and B(c, d), the midpoint M of

AB has coordinates M $\left(\frac{a+c}{2} \quad \frac{b+d}{2}\right)$.

Let the students do Exercise 10.2.

Magnitude of a vector

You need to help students be able to find the length or magnitude of a vector. Remind the students that every vector has a horizontal displacement and a vertical displacement. The components make up the column vector which when drawn gives a right angled triangle as in Fig 10.4 in the Student's Book. If need be, you can give them another example.

Ask the students to write down the column vector **PQ** and its magnitude, given that P is (a, b) and Q is (c, d). The length of a vector PQ is denoted as **|PQ|.** Students should be able to state that **|PQ|** = √V{ $(c – a)^2 + (d – b)^2$} .

Work out Example 10.3 with students and draw their attention to the highlighted note after it. Students should now do Exercise 10.3.

Discuss the students' responses to Question 4 and illustrate on the board.

Parallelogram law of addition

Explain to learners what it means to combine vectors linearly and how it results in solving simple equations. Use Examples 10.4 to illustrate this.

Note that only the very basic cases of linear combinations are considered.

Parallel vectors and collinear points

Remind students of scalar multiplication of vectors and that if one vector is a scalar multiple of the other, then the two vectors are parallel. The idea here is that the direction is the same but one vector is longer that the other.

When two vectors are parallel and they share a common point, then the two vectors must be on the same extended line. This idea may be used to test for collinearity of points. Explain how this is done, as is explained in the Student's Book using Fig. 10.8.

Lead students through Example 10.5 and ask them to do Exercise 10.4.

Discuss the students' responses to Question 7 and 8 to ensure that the students have their facts right.

Application of parallelogram law

Here, we establish by vector method well known geometrical results. As you take your students through Example 10.6, you should let them first state the properties of the shape. Then lead them through the process of establishing them using vector method.

Ask the students to do exercise 10.5.

This exercise is ideal for a class discussion. After the students have done their work, they need to know whether their reasoning was right or not. So, illustrate as much as is necessary.

Answers to exercises

Exercise 10.1 *(page 78–79)*

1. $OA = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $OB = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $OC = \begin{pmatrix} 1.5 \\ 3 \end{pmatrix}$ $OD = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$OE = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ $OF = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$ $OG = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$ $OH = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

$OI = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $OJ = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$ $OK = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ $OL = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$

3. (a) $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ (b) $\begin{pmatrix} 9 \\ 0 \end{pmatrix}$; $\begin{pmatrix} -9 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} -4 \\ -4 \end{pmatrix}$; $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

(d) (i) $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ (ii) $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$ (iii) $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ (iv) $\begin{pmatrix} -6 \\ -1 \end{pmatrix}$

(v) $\begin{pmatrix} 6 \\ 1 \end{pmatrix}$ (vi) $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$

4. (a) $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (c) $\begin{pmatrix} 7 \\ 12 \end{pmatrix}$

(d) $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (e) $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ (f) $\begin{pmatrix} -9 \\ -14 \end{pmatrix}$

Exercise 10.2 *(page 79)*

1. (a) (3.5, 2)

(b) (1, 5)

(c) (4, 1)

(d) (0, 2)

2. (a) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$; (3.5, 1.5)

(b) $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$; (1, 1)

(c) $\begin{pmatrix} -10 \\ -7 \end{pmatrix}$; (–7, –4.5)

(d) $\begin{pmatrix} 21 \\ -1 \end{pmatrix}$; (1.5, 0.5)

(e) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$; (–7.5, 7.5)

(f) $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$; (0, 0)

3. S(2, 2) ; (2.5, 1) ; (4.5, 3) ; (3.5, 3) ; (1.5, 1)

Exercise 10.3 *(page 80–81)*

1. (a) 5

(b) 10

(c) 13

(d) 8.062

2. (a) 6.403

(b) 10

(c) 6.403

(d) 7.616

(e) 5b

(f) 7.28 m

3. (a), (c)

4. (a) and (d) are true;

(b) and (c) are false.

5. (a) (i) 4.123

(ii) 3.606

(iii) 4

(b) ($^-$6, 2) ; (6, 2)

Exercise 10.4 *(page 82–83)*

1. (a) $\mathbf{a} = \frac{11}{14}\mathbf{b} + \frac{8}{7}\mathbf{c}$

(b) $\frac{14}{11}\mathbf{a} - \frac{16}{11}\mathbf{c}$

2. (a) $\mathbf{c} = {}^{-}\mathbf{a} + 4\mathbf{b}$

(b) $\mathbf{c} = \mathbf{a} - \frac{3}{2}\mathbf{b}$

3. (a) (i) $\mathbf{p} = \frac{-2}{3}\mathbf{q} + \mathbf{r}$

(ii) $\frac{-3}{2}\mathbf{q} = -\mathbf{p} + \frac{3}{2}\mathbf{r}$

(iii) $\mathbf{r} = \mathbf{p} + \frac{2}{3}\mathbf{q}$

(b) $\mathbf{q} = -\frac{k}{m}\mathbf{p}\ \frac{n}{m}\mathbf{r}$

4. $m = 2$, $n = 7$

5. $m = {}^{-}16$, $n = 42$

6. $\mathbf{m} = {}^{-}1$, $n = {}^{-}20$

8. Not collinear.

9. (a) (i) $\mathbf{p} + \mathbf{q}$

(ii) $\frac{1}{3}(\mathbf{p} + \mathbf{q})$

(iii) $\mathbf{q} - \mathbf{p}$

(iv) $\frac{1}{3}(2\mathbf{p} - \mathbf{q})$

(v) $\frac{4}{3}\mathbf{p} - \frac{2}{3}\mathbf{q}$

(b) The points are collinear

6–10 ANSWERS TO REVISION EXERCISES 2

(Student's Book pages 85–88)

Background knowledge

All that has been learnt in chapters 6 to 10.

Specific objectives

By the end of these revision exercises, the student should be able to answer similar exercise/questions accurately and with appropriate speed.

Teaching guidelines

See Revision exercise 1.

Revision Exercise 2.1 *(page 85)*

1. 11.7 cm^2

2. 13.8 cm^2

3. $h = 9$, $K = 16$

4. $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$; 3

5. (a) D (–7, 12)

(b) P(6, –4), Q(2.5, –5), R(–5, 6) S(–1.5, 7), T(0.5, 1)

(c) |AT| = 3.64, |CT| = 3.64

(d) SR = PQ; SP = RQ, thus PQRS is a parallelogram.

6. $-7x^5 + 16x^3 - 2x^2 - 7x + 27$

7. Quotient: $\times^2 - 9x - 27$
Remainder: –123

8. $x^5 + 6x^4 - 10x^3 - 36x^2 + 24x$ degree is 5

9. (a) 0.63

(b) 0.03

(c) 0.34

(d) 0.97

10. (a) 0.009

(b) 0.407

(c) 0.108

11. D(2, –1)

12. (a) 1 m/s

(b) 1 m/s

(c) 17.5 m

Revision Exercise 2.2 *(page 86)*

1. 160.58 cm^2

2. $\angle A = 82.8^\circ$, $\angle = 41.4^\circ$, $\angle C = 55.8^\circ$

3. (a) $\boldsymbol{e} = \frac{3}{5}b + \frac{2}{5}c$, $\boldsymbol{f} = \frac{3}{7}a + \frac{4}{7}\text{c}$, $\boldsymbol{g} = 2b - a$,

(b) $\boldsymbol{FE} = \frac{3}{7}\boldsymbol{EG}$, hence E,F and G are collinear.

4. 3

5. (a) $\begin{pmatrix} 0 \\ 4.4 \\ -6 \end{pmatrix}$

(b) 7.44

6. Quotient: $x^2 - 1$
Remainder: –4

7. $4x^5 + 4x^4 - 4x^3 + 6x^2 - 6$

8. Quotient: $x^2 - x - 3$
Remainder: –1

9. (a) $\frac{11}{12}$

(b) $\frac{2}{9}$

(c) $\frac{3}{4}$

10. (a) $\frac{1}{2}$

(b) $\frac{11}{20}$

(c) $\frac{9}{20}$

11. (a) $\frac{1}{64}$

(b) $\frac{27}{64}$

(c) $\frac{27}{64}$

12. $v = \frac{t^2}{2} - 6t + 2$, $s = \frac{t^3}{6} - 3t^2 + 2t$

Revision **Exercise 2.3** *(page 86–88)*

1. (a) 33.75°

(b) 66.42°

2. 19.4 km, 255°

3. (a) (i) 4**q** – 12**p**

(ii) 3**q** – 9**p**

(iii) ;3**p** – 3**q**

(iv) 3**q**

(b) h = – 8 – a
K = 3a

4. (b) (–4, –11)

(c) $m = \frac{85}{14}$, $n = \frac{13}{7}$

5. (a) $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$

(b) 17.889

6. (a) degree 6

(b) degree 3

7. $x^2 + 5x$

8. 1

9. 0.521 4

10. $\frac{230}{319}$

11. (b) (i) 8 m/s^2

(ii) 0 m/s^2

(iii) –1.6 m/s^2

(c) 384 m

12. (a) $\frac{1}{10,000}$

(b) 0.656 1

(c) 0.082

11 LINEAR PROGRAMMING

(Student's Book pages 89–96)

Background knowledge

Solution of linear inequalities, forming simple inequalities from given graphs and narratives, graphical representation and solution of linear inequalities, finding equations of lines.

Objectives

By the end of this chapter, the student should be able to:

(a) identify variables,

(b) formulate inequalities,

(c) formulate objective function,

(d) illustrate graphically the region described by inequalities (shading the unwanted region),

(e) find solutions of a linear programming problem using the graph and the objective function.

Subtopics

- Graphical solution of linear inequalities.
- Variables.
- Maximising or minimising a function.
- Objective function.

Suggested resources

- Square boards
- Graph books/papers

Teaching guidelines

Review of graphical solution of linear inequalities

Remind the students the procedure of representing linear inequalities graphically, the meaning of broken and solid lines and the process of identifying the solution set (required region). Help them work through Example 11.1 and 11.2 before they do Exercise 11.1.

Variables

The bulk of work in this section is revision. In Book 3, students learned how to form inequalities from word problems and from inequality graphs. As revision, take them through Examples 11.3 then ask them to do Exercise 11.2.

Maximising or minimising a function.

In this section the objective is to help students determine the maximum or minimum value of a specific function in a given region. In order to be able to demonstrate how to do this effectively, copy Fig. 11.3 on the board, where every student can see it. Explain how you get the equation of the search line from the given function and draw it on the same graph. Then, working on the board, demonstrate how to maximise or minimise, as described in Example 11.4. Encourage the students to do the same in their graph books as you go round the class helping those who may have difficulties with the process.

Students should then do Exercise 11.3

Objective function

Explain the meaning of the term optimisation, relating it to maximising and minimising of a function. Let students give you examples from real life situations where maximisation or minimisation may be applicable. Explain clearly, the meaning of objective function. Illustrate Example 11.5 on the board.

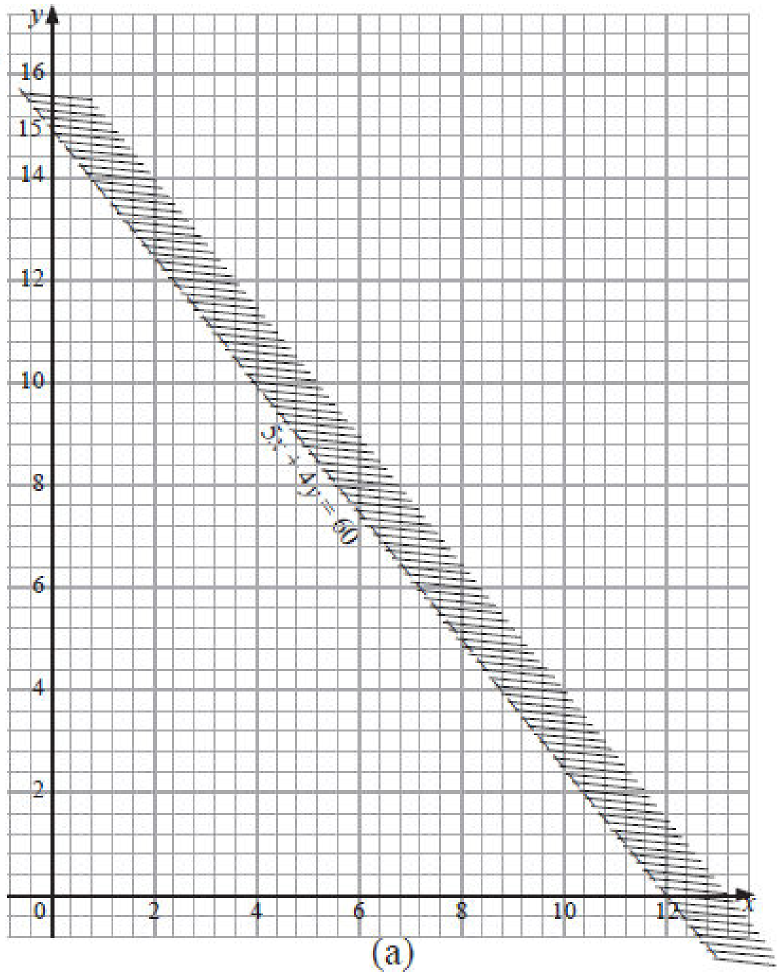
Ask students to do Exercise 11.4.

In this exercise, all the questions involving graphs should be illustrated on the chalk board for all to see and discuss if need be.

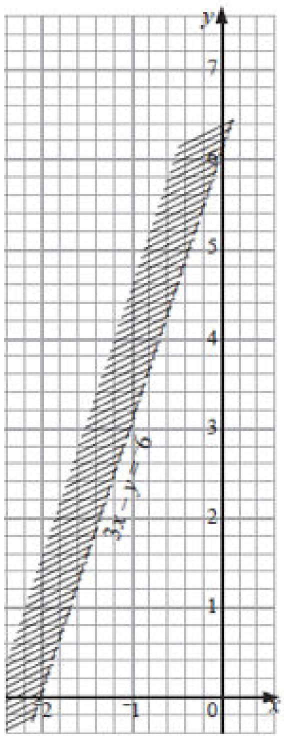
Answers to exercises

Exercise 11.1 *(page 90)*

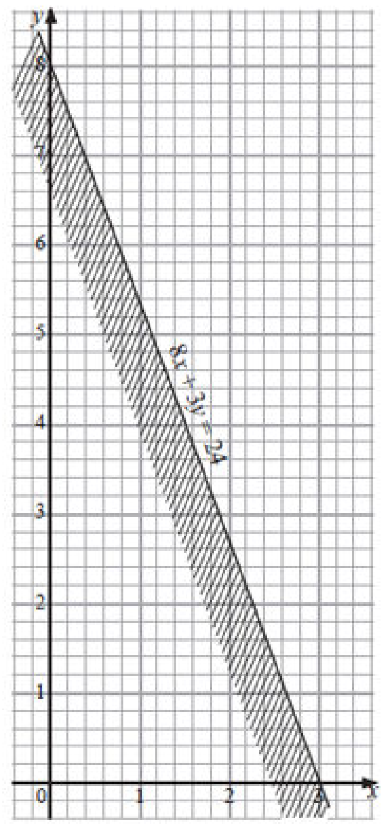
1.



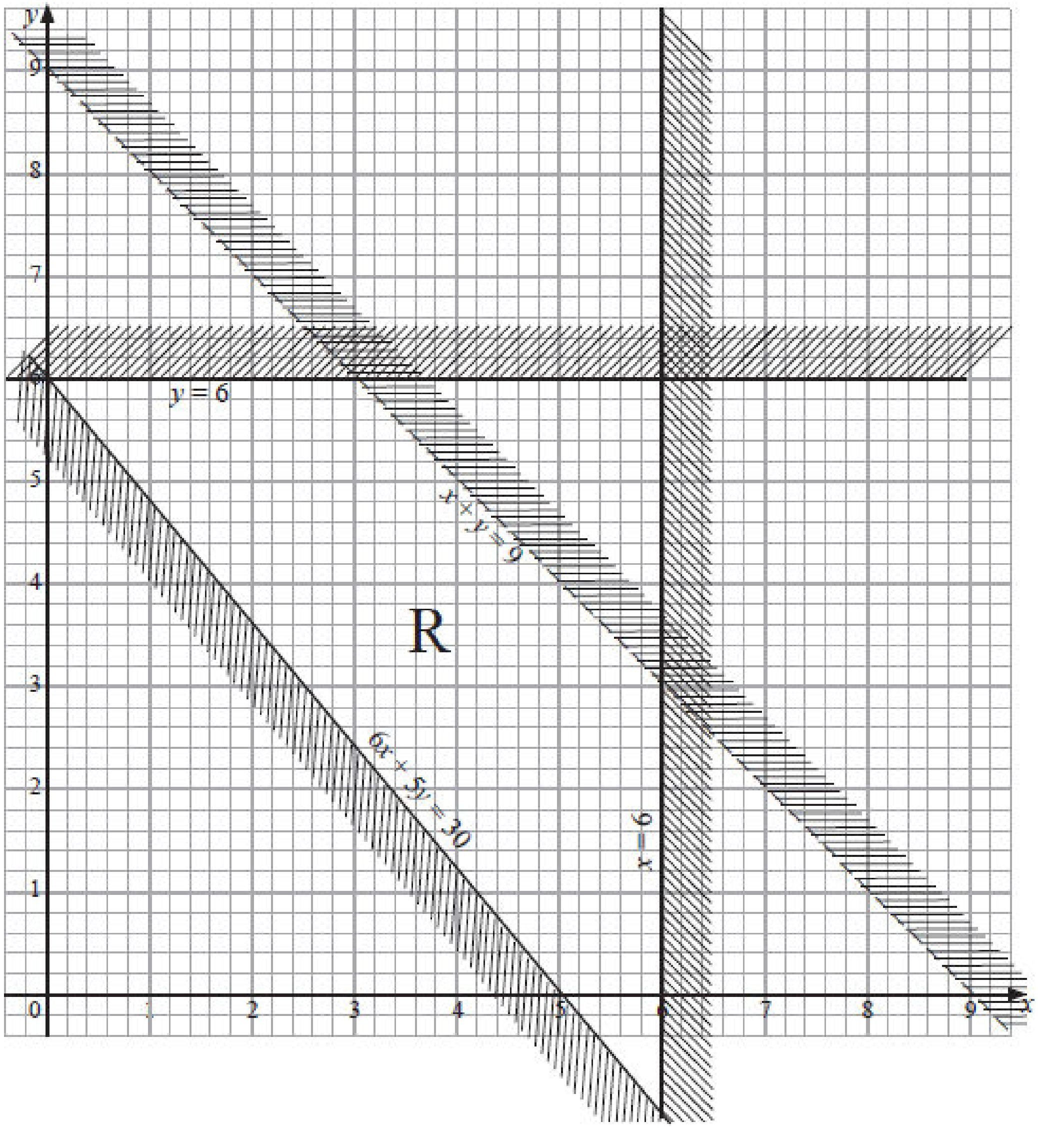
(a)



(b)



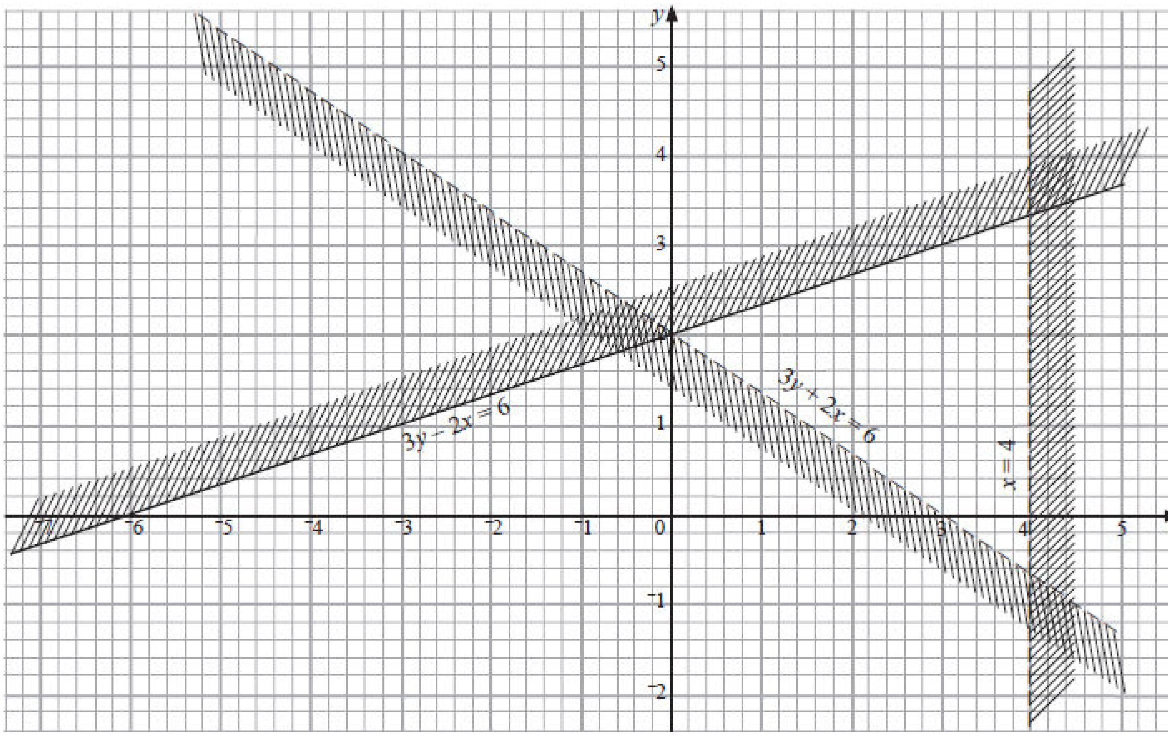
(c)



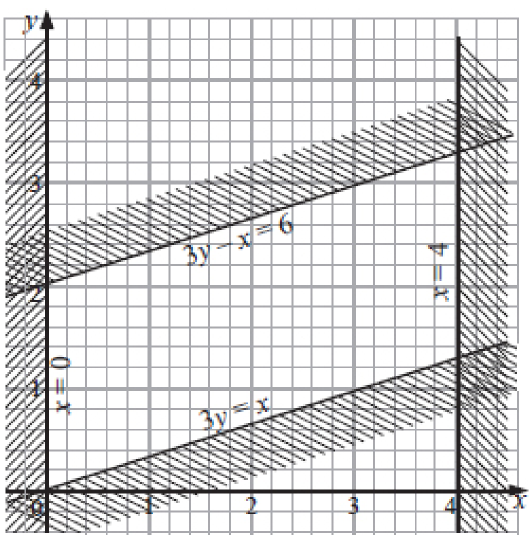
2.

(0, 6), (1, 5), (1, 6),
(2, 4), (2, 5), (2, 6),
(3, 3), (3, 4), (3, 5),
(3, 6), (4, 2), (4, 3),
(4, 4), (5, 0), (5, 1),
(5, 2), (5, 3), (6, $^{-}$1)
(6, 0), (6, 1), (6, 2).

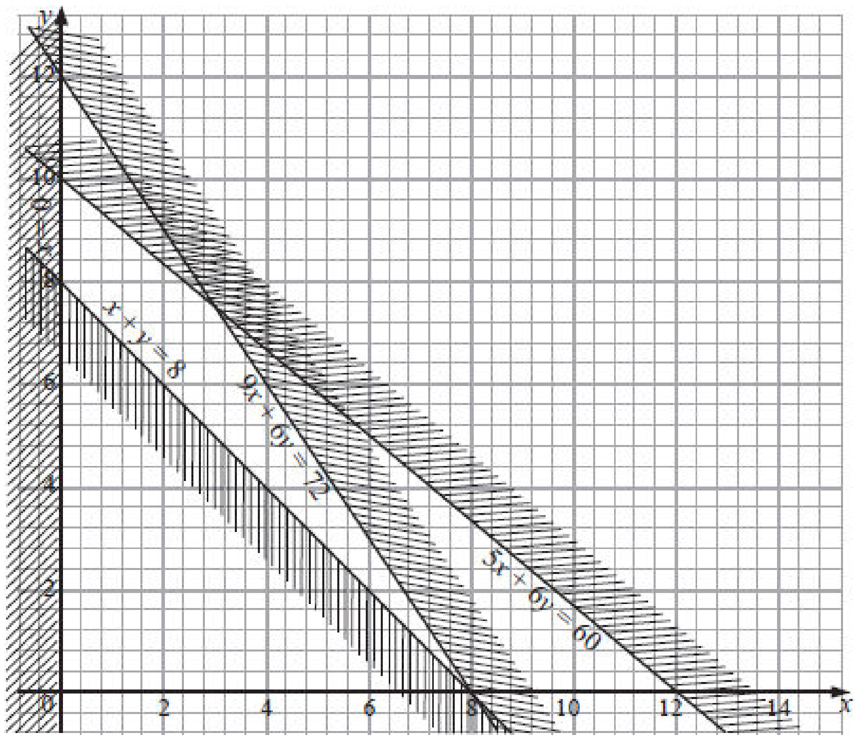
3.



(a) (1, 2), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)



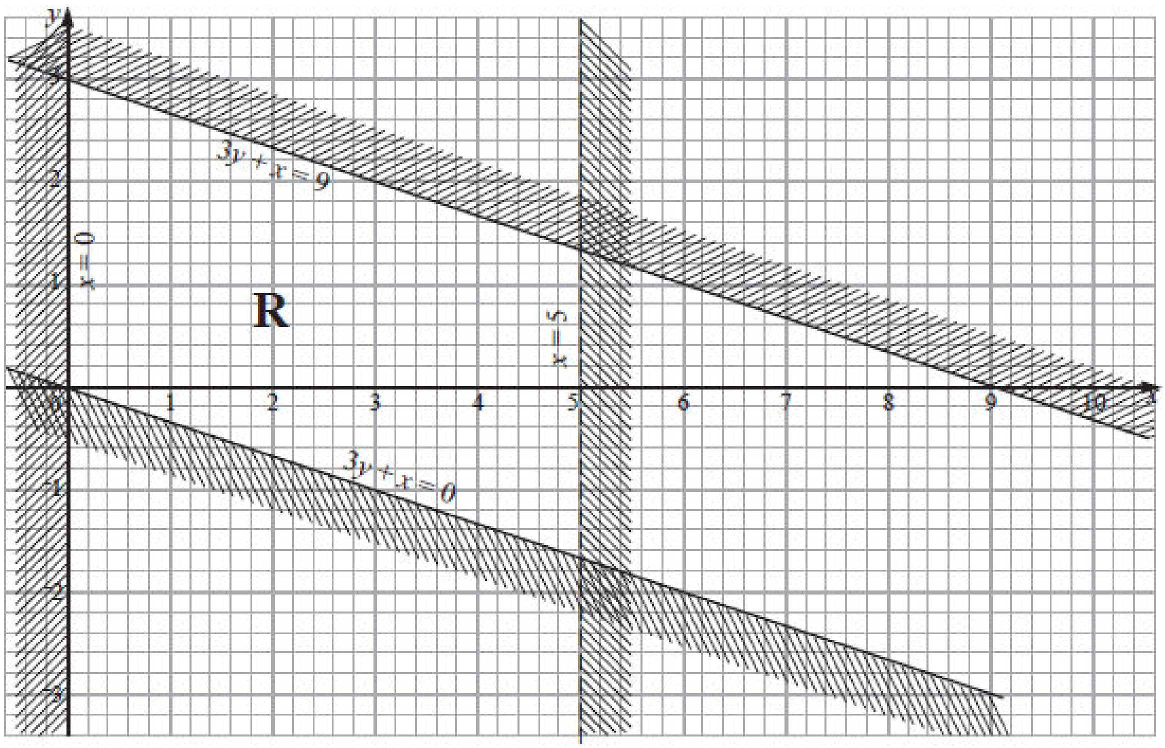
(b) (1, 1), (1, 2) (2, 1), (2, 2) (3, 2)
(0, 0) (0, 1) (0, 2), (3, 1) (3, 3)
(4, 2) (4, 3)



(c) (0, 8), (0, 9), (0, 10), (1, 7), (1, 8), (1, 9), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (6, 2), (6, 3), (7, 1), (8, 0)

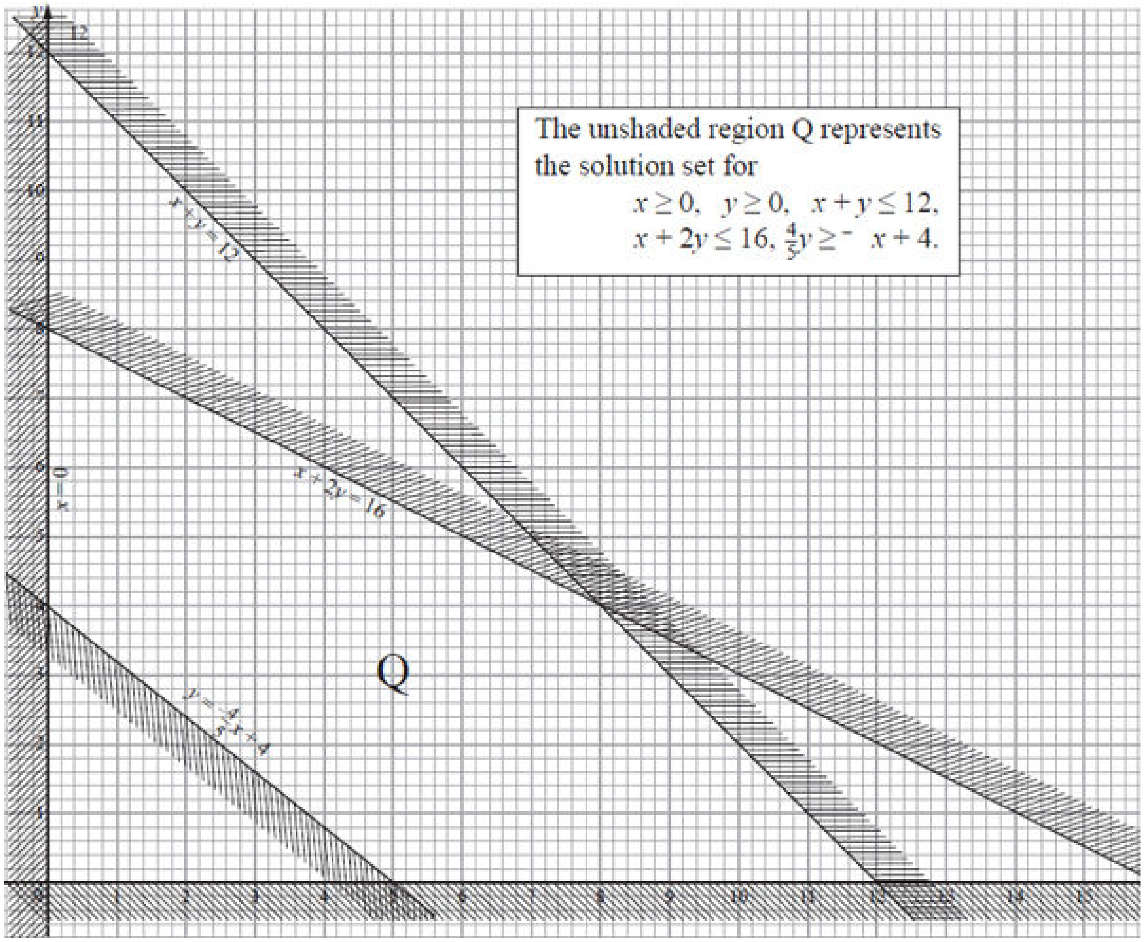
4. (3, 0), (4, 0), (2, 1), (3, 1), (4, 1), (3, 3), (4, 3), (0, 2), (1, 2), (2, 2), (3, 2), (4, 2)

5.



The unshaded region R represents the solution set for the inequalities $0 \leq x < 5$ and $0 \leq 3y + x \leq 9$.

6.



Exercise 11.2 *(page 91)*

1. Let x be number of cards at K 60.
   Let y be number of cards at K 90.
   Inequalities are:
   $x \geq 0$, $y \geq 0$, 60x + 90y ≤ 540, $x + y \geq 4$

2. Let x be number of type A aircrafts.
   Let y be number of type B aircrafts.
   Inequalities are:
   $x + y \leq 15$, $60x + 70y \geq 600$, $6x + 3y \geq 45$

3. Let x be number of trucks available.
   Let y be number of vans available.
   Inequalities are:
   $x > 0$, $y > 0$, 500x + 400y ≤ 4 400, $150x + 60y \geq 900$.

4. Let a be the number of type A fireworks bought.
   Let b be the number of type B fireworks bought.

Inequalities are: $60a + 90b \leq 720$, $a \geq 2b$.

5. $80x + 25y \leq 400$, $x < y$, $x \geq 0$, $y \geq 0$

6. Let x be number of layers
Let y be number of broilers
$65x + 45y \leq 40\ 000$, $x + y \geq 400$, $y > x$, $y \geq 300$.

7. $(n + 1)$, $(n + 2)$, $3n + 3 < 999$

8. $5 + 9 > x$, $5 + x > 9$, $9 + x > 5$

Exercise 11.3 *(page 92–93)*

1. (a) 6

(b) 15

2. (a) 13.8

(b) 13.5

(c) 9.4

3. (a) a/min = 1

(b) b/min = 16

(c) c/min = 16

4. $5x + 4y \geq 50$, $5x + 3y \geq 42$, $x \geq 0$, $y \geq 0$
Min. value = 10 800.

5. (a) $5l + 2v \geq 30$, $l > 0$, $v > 0$

(b) $5l + 4v \leq 44$

(d) (i)

l	6	7	6	7	5	6	5	4
v	1	1	2	2	3	3	4	4

(ii) 4 lorries and 5 vans ; cost is K 40 000

(iii) 7 lorries and 2 vans ; cost is K 43 000

Exercise 11.4 *(page 94–96)*

1. $300x + 500y \geq 150\ 000$
$y \geq 2x$, $x + y \leq 600$, $x \geq 0$, $y \geq 0$

(b) K 300 000

(c) K 288 000

2. $x + y \leq 18$
$1\,000x + 800y \leq 12\,000$, $2x + y \leq 18$.
$x \geq 0$, $y \geq 0$
Maximum profit = K 25 600 by planting 4 ha with beans and 10 ha with potatoes.

3. (a) $500x + 600y \leq 4\,500$, $9x + 6y \leq 54$, $x > 0$, $y > 0$

(b) All points inside the unshaded region and along the boundary whose coordinates are whole numbers.

(c) (i) 4 machines of type A and 3 of type B (1 800 units).

(ii) 4 machines of type A and 3 of type B.

4. (a) $x + 2y \leq 28$, $3x + y \leq 24$, $x \geq 0$, $y \geq 0$

(c) 4 tables and 12 chairs

(d) (i) K 7 000

(ii) K 6 400

(iii) K 7 800

(iv) K 9 200

5. (a) $5x + 2y \geq 80$, $3x < 2y$, $x \geq 0$, $y \geq 0$

(c) Objective function: $20x + 5y = 100$. Cheapest combination: 8 mangoes for every 20 oranges

6. (a) $3x + 4y \leq 36$, $x > y$, $x > 3$, $y \geq 2$

(c) Possible song combinations:

Classicals	4	5	6	7	8	9	4	5	6	7	8	5	6
Raps	2	2	2	2	2	2	3	3	3	3	3	4	4

7. (a) $3x + 0.5y \leq 40$, $x \geq 0$, $y \geq 0$, $2x + 5y \leq 45$

(b) 13 of type A and 4 of type B.

8. (a) $2x + 3y \geq 120$, $y \leq 2x$, $x + y \leq 60$

(b) Objective function: $x + 2y = c$.
For maximum profit: $x = 20$, $y = 40$.

12 THREE DIMENSIONAL FIGURES

(Student's Book pages 97–120)

Background knowledge

Geometric constructions, areas of plane figures, properties of common solids, volumes and surface area of common solids, angles, Pythagorus theorem, trignometric ratios similarity.

Objectives

By the end of this chapter, the student should be able to:

(a) sketch three dimensional figures,
(b) find surface areas of 3-D figures,
(c) find volumes of 3-D figures,
(d) identify vertical, horizontal and slanting lines (edges) and planes,
(e) identify angles between two lines,
(f) identify angles between two planes,
(g) identify angles between a plane and a line,
(h) calculate lengths of sides in 3-D figures,
(i) calculate angles in 3-D figures.

Subtopics

- Sketching solids.
- Surface area of solids.
- Volume of solids.
- Points, lines (edges) and planes.
- Angles between two planes.
- Angles between two lines.
- Angles between a line and a plane.

Suggested resources

- Geometrical instruments.
- Models of solids - prism, pyramid, cone, frustrum, sphere.
- Skeletons of 3 dimensional solids.
- Detachable models.

Teaching guidelines

Effective teaching of this topic can only take place if the teacher is familiar with model making and is good at drawing three dimensional diagrams. Students will need a lot of guidance in making, drawing and using the three dimensional models to solve problems.

Using a simple common solid like a rectangular box, help students to identify and define edges, vertices and faces.

Sketching solids

For sucessful sketching of solids, explain to the students that:

1. Vertical and parallel edges must be represented as vertical and parallel lines.
2. Horizontal and parallel edges must be repsented as horizontal and parallel lines.
3. Vertical and horizontal faces must be represented as such in the diagram.

Other facts and observations will follow later as we work through the recommended activities. Using Fig. 12.1, let the students see that faces that are directly opposite to you are drawn in their true shapes and sizes, while all others take the shape of a parallelogram. These observations are more meaningful when they come from the students themselves. Take the students through activities 12.1 and 12.2, illustrating the procedure on the chalkboard. Ensure that every student does the activity in their exercise books. At the end of the activity, help the students to come up with answers to the questions at the end. Advise your class that faces that seem to be distorted in the sketch have to be so, otherwise the solid will not look like one. Also, edges that are perpendicular to the front face (Fig. 12.3) are normally drawn about half their actual length and at an angle of about 30° to 45°.

Ask students to do Exercise 12.1

Illustrate Questions 1 and 4 of Exercise 12.1 on the chalk board. This will help the students assess themselves.

Surface area of Prisms

Ensure that students are able to distinguish between a prism and any other solid. Let them identify the properties of all types of prism, including their symmetry. Remind them that a prism is named after the shape of its cross-section.

Surface area of a cone

Activity 12.3 in the Student's Book is meant to help students derive the formula for finding the curved area of a cone. Work through the activity in advance to ensure that the expected conclusions are arrived at, and also to be able to help your students should there be need to. Then guide them through Example 12.3 and let them do Exercise 12.4 thereafter.

In this section, your students will need the knowledge of similar triangles and how to use Pythagoras' theorem in calculating lengths. It is therefore recommended that you begin by revisiting work on similar triangles and Pythagoras' theorem before going into the work on frustums.

Using clearly labelled diagrams on a chart or board, take students through Example 12.4. Now, let students do Exercise 12.5.

Surface area of a sphere

In this section, just ensure that students use the correct formula. Work through Example 12.5 and let the students do Exercise 12.6.

Volume of a prism

In Book 2, we learnt how to find volume of a cylinder and some other solids of uniform cross-section. Remind the students some of the solids dealt with. In Book 2 prisms were defined. Let the learners name some common prisms. Use models of prisms of different cross-sections. Through discussions, remind students that for any prism,

volume = area of uniform cross-section × length (or height)

Use Example 12.5 to illustrate how to calculate the volume of a prism. It is important that students identify the correct cross-section area. Let students do Exercise 12.7.

Volume of a pyramid

Discuss with students what a pyramid is and give examples. Use models to assist them to concretise the concept of a pyramid. Note that students may not be too familiar with pyramids in their everyday life. Hence ensure that you have good solid models as well as their nets and diagrams.

Let all learners do Activity 12.4. Proper supervision is required to ensure that each student gets the correct expected outcomes. You should have your model of Activity 12.4 to help you assist the students to see that:

$$\text{volume of a pyramid} = \frac{1}{3} \times \text{volume of cube it was cut from}$$

Using the volume of a cube as 'base area x height' assist students to see that:

$$\text{volume of a pyramid} = \frac{1}{3}Ah$$

where A = area of base

and h = vertical height of the pyramid.

Use Example12.6 to illustrate how to find the volume of pyramids then ask students to do Exercise 12.8.

Volume of a cone

Using a model of a cone, show the students how a cone may be regarded as a right pyramid with a circular base. Then deduce that:

$$\text{volume of a cone} = \frac{1}{3} \times \text{base area} \times \text{height}$$
$$= \frac{1}{3}\pi r^2 h$$

Use a drawing of a cone and a model of the same to identify the slanting side l, the height h and the radius r. Since all these form a right angled triangle, let the students use the Pythagoras' theorem to get the relation.

$$l^2 = h^2 + r^2$$

Use Example 12.7 to illustrate working out of volume of a cone and let students do Exercise 12.9.

Use models of a cone and a pyramid. Show what happens when you cut off the top along a plane parallel to the base. Use models which have been cut so that you just lift off the top to leave the frustum. Let the students see and remember the solid that remains – called a frustum. Use models and charts to help students see how you reconstruct the original cone or model from a given frustum.

Through discussion with students, state the steps followed in calculating the volume of a frustum as given in Student's Book. Discuss an example such as Example 12.8 on how to calculate volume of a frustum. Emphasize that similarity is used in finding the volume of a frustum.

Let students do Exercise 12.10

Volume of a sphere

When deriving the formula of volume of a sphere, make use of the volume of a pyramid. As can be seen from Fig 12.37 in Student's Book, if a very small piece of the surface of a sphere is cut out as a small square, it would be flat.

When the vertices are joined to the centre of a sphere, we get a small pyramid of volume = $\frac{1}{3}Ar$, where r is radius of the sphere and A is base area of the small pyramid. Let the students see that if there are many such small '*pyramids*', adding all their base areas is equivalent to getting the surface area of a sphere, which is $4\pi r^2$. It then follows that the total volume of a sphere becomes;

$$\frac{1}{3}r \times 4\pi r^2 \text{ i.e. } V = \pi r^2$$

Use Example 12.9 to illustrate the calculation of volume of a sphere. Students may now do Exercise 12.11.

Points, lines and planes

Explain these concepts to students, as is done in the Student's Book, so that they are familiar with their usage in three dimensional geometry.

Identification of a plane

Explain to students the fact that any three points, in space, that are non-colinear are coplanar, i.e. they determine a plane. Also, two parallel or intersecting lines determine a plane. You may use different models to illustrate these facts and help students to make appropriate sketches of planes.

Highlight the fact that two skew lines do not determine a plane. Show this using appropriate models.

Let students consolidate this by doing Exercise 12.12.

Projections and angles

Projections

Use students' experiences of shadows to explain the concept of "projections". You may use Fig. 12.45 in the Student's Book to further clarify the concept, highlighting the difference between "meeting the ground normally" and meeting the ground obliquely". Let students identify projections of points and of lines.

Angle between two lines

Explain to the students that the angle between two lines is always the acute angle at the point of intersection of the two lines. Make the students see that translating one of the lines does not change the size of the angle between the lines. Do this using the property of corresponding angles on parallel lines. This is the idea used to define the angle between two skew lines.

Ask students to do Exercise 12.13.

Angle between a line and a plane

Let students carry out Activity 12.5 to be able to differentiate cases of a line being perpendicular to a plane and those where a line meets the plane obliquely. Explain to the students that the angle between a line and a plane is defined as the angle between the line and its projection on the plane. You could illustrate the projection of a line onto a plane using a wire frame model by shining a torch from directly above the chosen line.

Angle between two planes

Guide students through Activity 12.6 to identify the angle between two planes. You need to emphasise that the lines, one on either plane, that determine the angle between the two planes, must meet the line of intersection of the planes at right angles.

Let students now do Exercise 12.14.

Calculating lengths and angles in solids

The important thing to emphasise here is that in three dimensions, unknown angles can, in most cases, be determined by solving right-angled triangles. Thus, the knowledge of Pythagoras' theorem and basic trigonometric ratios is called for.

Insist on the extraction of the relevant triangles (and sketching them) to show where they are right-angled, in order to reduce the chances of making mistakes. Examples 12.10 to 12.11 in the Student's Book, elaborately illustrate the process. Take the students through them, and ensure that you explain the idea of the shortest distance between a line and a plane (as in Example 12.10 (c)).

Similarly, pay attention to the calculation of the angle between two planes that meet along a line that is oblique to the horizontal base of a pyramid (Example 12.11), as this is probably the most difficult angle for most students: It is one of the few cases where we use Pythagoras' theorem, area of a triangle calculated in two ways, and the cosine rule of solving triangles.

Ask students to do Exercise 12.15.

Answers

Exercise 12.1 *(page 98–99)*

1. (a) Parallelogram, rectangle

(b) ABFE and CDHG

(c) EH, AD, BC; yes

(d) BD, FH

2. (b) prism

(c) They are equal

(d) PU, PT

(e) Yes; SU, PR, PQ, SR, SQ, ST.

Exercise 12.2 *(page 101)*

1. (a) 136 cm^2

(b) 120 cm^2

(c) 437.66 cm^2

(d) 1 155 cm^2

(e) 551.3 cm^2

(f) 691.08 cm^2

2. 378π

3. 462 cm^2

4. 11 cm

Exercise 12.3 *(page 101)*

1. 96 cm^2

2. 77.63 cm^2

3. 61.6 cm^2

4. 472.98 cm^2

5. (a) 568.32 cm^2

(b) 203 cm^2

Exercise 12.4 *(page 103)*

1. 263.9 cm^2

2. 942 cm^2

3. 301.4 cm^2

4. 108.7 cm^2

5. 240.4 cm^2

6. 65.47 cm^2

7. 46.29 cm^2

8. (a) 15 cm^2

(b) 17.87 cm^2

9. 30.65 m^2

Exercise 12.5 *(page 104)*

1. 162.6 cm^2

2. 2 802 cm^2

3. 171.2 cm^2

4. 930.03 cm^2

5. 2 973 cm^2

Exercise 12.6 *(page 105)*

1. (a) 128.7 cm^2

(b) 18.10 cm^2

(c) 221.7 cm^2

2. (a) 2.50 cm

(b) 3.79 cm

3. 235.65 cm^2

4. 1 134 cm^2

5. 1 034 cm^2

Exercise 12.7 *(page 106–107)*

1. (a) 96 cm^3

(b) 54 cm^3

(c) 429.3 cm^3

(d) 2 425.5 cm^3

(e) 735 cm^3

(f) 218.3 cm^3

2. 134 400 cm^3, 0.134 4 m^3

3. 11 644 cm^3

4. (a) 15 600 cm^3

(b) 16 800 cm^3

(c) 28 800 cm^3

5. 3.08 liters

6. 0.7 m

7. 45 cm

8. 4.47 cm

Exercise 12.8 *(page 108)*

1. 48 cm^3

2. 162 cm^3

3. 40 cm^3

4. 40 cm^3

5. 128 cm^3

6. 174.6 cm^3
7. 51.96 cm^3
8. 15.08 cm^3
9. 70.67 cm^3
10. 18 cm^2
11. 12 cm
12. 111.9 cm^3
13. (a) 832 cm^3
    (b) 183.3 cm^3

Exercise 12.9 *(page 108–109)*

1. 20 cm^3
2. 199.4 cm^3
3. 754.1 cm^3
4. 301.6 cm^3
5. 75.41 cm^3
6. 204.1 cm^3
7. 33.6 cm^3
8. 9.87 cm
9. 10 cm, 1.592 cm, 9.736 cm^3
10. 19.63 cm^3
11. 102.6 cm^3

Exercise 12.10 *(page 109–110)*

1. 140 cm^3
2. 18.99 *l*
3. 130.3 cm^3

4. 65.975 cm^3

5. 1 892 cm^3

Exercise 12.11 *(page 110–111)*

1. (a) 137.3 cm^3

(b) 7.239 cm^3

(c) 310.4 cm^3

2. (a) 2.60 cm

(b) 4.80 cm

3. (a) 9.182 cm^3

(b) 2 849 cm^3

4. 523.3 cm^3

5. 323.4 g

6. 537.6 cm^3

7. 3.016 cm

8. 8.6 cm^3

9. (a) 532.2 cm^2

(b) 6.9 cm

Exercise 12.12 *(page 112–113)*

1. (a) AD, AE, CD, ED, EF, EH, DH

(b) PT, RT

2. BF, DH

3. (a), (c), (d), (f), (h)

4. (a), (c), (e), (f), (h), (k), (l)

5. Yes ; G ; Triangular prisms

6. Square, frustum.

Exercise 12.13 *(page 114)*

1. (a) $\overline{AO}$

(b) $\overline{VO}$

(c) $\overline{VO}$

(d) $\overline{CO}$

2. (a) $\overline{AC}$

(b) $\overline{AH}$

(c) $\overline{FH}$

(d) $\overline{CH}$

(e) $\overline{BD}$

(f) $\overline{FC}$

(g) $\overline{HG}$

(h) $\overline{AB}$

3. (a) $\angle VCD$

(b) $\angle VAB$

(c) $\angle VAD$

(d) $\angle VCB$

4. (a) 45°

(b) 45°

(c) 90°

(d) 45°

(e) 0°

(f) 90°

(g) 0°

(h) 60°

5. AD, AE, BC, BF

6. AB, AD, BC, DC

7. (a) BC, CF, DC

(b) (i) ∠DAE and ∠CBF

(ii) ∠CF and ∠ADE

(iii) ∠AC and ∠ACD

Exercise 12.14 *(page 116)*

1. (a) ∠VQO

(b) ∠OVR

(c) ∠OVS

2. (a) ∠ACE

(b) ∠DFB

(c) ∠CF

(d) ∠AHD

(e) ∠EBQ

(f) ∠GP

3. (a) 90°

(b) 0°

(c) 45°

(d) 0°

4. (a) AD

(b) BF

(c) CG

(d) PQ

5. (a) QS, 90°

(b) OV, 90°

6. (a) 90°

(b) 90°

(c) 45°

(d) 90°

7. ∠VMO where M is the midpoint of QR

Exercise 12.15 *(page 119–120)*

1. 14.68

2. 56.79

3. (a) 21.80°

(b) 47.97°

(c) 67.38°

(d) 42.03°

(e) 26.57°

4. (a) 16.5 cm

(b) 55.55°

(c) 64.1°

5. (a) 60.9°

(b) 75.06°

6. (a) 70.53°

(b) 54.73°

7. 97.2°

8. (a) 7.07 cm

(b) 53.13°

(c) 3.12 cm

13 GRAPHS OF CUBIC FUNCTIONS

(Student's Book pages 121–127)

Background knowledge

Graphical solution of linear and quadratic equations, gradient of a straight line, definition and properties of a circle, use of Pythagoras' theorem, completing the square.

Objectives

By the end of this chapter, the student should be able to:

(a) construct table of values,

(b) draw graphs of cubic functions,

(c) solve cubic equations graphically,

(d) solve the simultaneous linear and cubic equations graphically.

Subtopics

- Tables of values and graphs of given relations.
- Graphs of cubic relations.
- The graph of $y = x^3$.
- The graph of $y = ax^3 + bx^2 + cx + d$.
- Solving cubic equations.
- Simulteneous Equations: One linear one cubic.

Resources

- Graph papers.
- Square boards.
- Geometrical instruments.
- Charts illustrating tangents to a curve.

Teaching guidelines

Tables of values and graphs of given relations

The students should be familiar with linear and quadratic relations. They have also made and used tables of values to draw linear and quadratic graphs. Introduce this chapter by briefly revising the graphical work done so far.

Take students through Example 13.1 emphasising the method of forming a table of values by breaking down the expression into its components. Questions that involve groups should be illustrated on the chalk board.

Ask the students to do Exercise 13.1.

Graphs of cubic relations

Define a cubic relation emphasising the basic requirement for a relation to be cubic. For example,

(i) $y = ax^3$
(ii) $y = ax^3 + d$
(iii) $y = ax^3 + bx^2$
(iv) $y = ax^3 + cx + d$
(v) $y = ax^3 + bx^2 + cx$
(vi) $y = ax^3 + bx^2 + c\,x + d$

(where *a, b, c, d* are constants and *x* is the variable) are all cubic relations. The term in x^3 is the determining factor.

The graph of $y = x^3$

Students should be able to make a table of values for $y = x^3$. Guide them through Example 13.2, emphasising the given procedure of drawing the graph and interpreting it.

The graph of $y = ax^3 + bx^2 + cx + d$

The method used to draw the graph of $y = ax^3 + bx^2 + cx + d$ is similar to that used in drawing a quadratic graph. Table 13.6 illustrates a step by step method of making a table of values by breaking the relation into individual terms. This method helps to minimise possible errors. Insist that your students use the method as given in Example 13.2 whenever they are required to make a table of values for a relation involving two or more terms. Also, discuss the characteristics and properties of the general cubic curve as illustrated in Fig. 13.4 in the Student's Book. Ask Students to do exercise 13.2.

Solving cubic equations

Solving cubic equations graphically is similar to solving quadratic equations. Just as a quadratic equation has a maximum of two real roots, a cubic equation has a maximum of three real roots.

The roots of a cubic equation are found at the point where the graph meets the *x*-axis. The graph of a given cubic relation may be used to solve related equations as shown in Example 13.3. Take students through this example and then ask them to do Exercise 13.3.

Simulteneous equations: One linear, one cubic.

We can use graphs to solve simulteneous equation one cubic and one linear. Guide the students in plotting the values of each equation by first making a table as shown in example 13.4. Guide them in finding the values of y and plot the values as shown in Fig 13.7. The point where the two graphs intersects is the solution of the simulteneous equations.

Ask the students to do Exercise 13.3.

Answers to exercises

Exercise 13.1 *(page 122)*

1. (a)

x	$^-3$	$^-2$	0	1	2	3	5
y	8	7	5	4	3	2	0

(b)

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	8	3	0	$^-1$	0	3	8

2. $x = 5.6$ $y = 31.5$; $x^3 - 25x = 36$

3. (a) 0.7 cm 5.3 cm

Exercise 13.3 *(page 127)*

1. (a) 1.12

(b) 1.2

2. (a) 1

(b) $^-1$

3. 2.8

4. (a) 4.5

(b) $^-0.9$; 1.2 ; 3.7

(c) $^-\frac{1}{2}$; 0.8 ; 3.9

5. (a) 0.3 ; 1.4

(b) 0.3 ; 1.4

6. 4.6, 5.8, 3.6 ; $\frac{1}{4}x^3 + 5x - 30 = 0$

7. (a) 1.15; 3.7

(b) $^-0.75$; 1.1 ; 3.6 ;

(c) $x^3 - 4x^2 + {}^-\frac{1}{2}x + 3 = 0$; $y = {}^-\frac{1}{2}x + 4$

8. $x = 1.111$, when $y = 0.55$
$x = 3.64$, when $y = 4.32$
$x = 0.7$, when $y = 2.65$

9. $x = -0.17$, when $y = 0.65$
$x = 1.64$, when $y = -2.14$

10. $x = 2.9, -0.3, -2.7$

11–13 ANSWERS TO REVISION EXERCISES 3

(Student's Book pages 128–132)

Background knowledge

All that has been learnt in chapters 11 to 13.

Specific objectives

By the end of these revision exercises, the student should be able to answer similar exercise/questions accurately and with appropriate speed.

Teaching guidelines

See Revision exercise 1.

Revision Exercise 3.1 *(page 128–129)*

1. (a) 0, 1, 2

(b) 2.7

2. (a) $x = 2.11$ or 0.25 or $–1.86$

(b) $0.1x^3 – 0.4x – 0.1 = 0$

or

$x^3 – 4x – 1 = 0$

3. (a) 3 cm

(b) 37.7 cm^2

4. (a) 1 m

(b) 26.57°

(c) 25.84°

5. (a) AE, DH, EF, EH, HG

(b) BC, BF, CG, FG

(c) (i) and (iii)

(d) (i) AC

(ii) BC

6. (a) 23.5 cm

(b) 69.87°

(c) 73.4°

(d) 78°

7. $y \geq -\frac{3}{5}x$, $y \geq 2x$, $7y – x < 26$

8. (a) $x > -\frac{3}{5}$

(b) $x < -\frac{3}{2}$

(c) $x \geq \frac{-2}{3}$

9. (a) (3, 2), (4, 0), (4, 1), (5, 0), (5, 1), (5, 2), (6, 0), (6, 1), (7, 0)

(b) $\frac{-8}{3}$

(c) 24

10. $x + y \leq 70$, $x \geq 10$, $y \geq 20$,
$x + 2y \leq 120$, $3x + y \leq 180$

11. $a = 1.37$, $c = -2.7$, $y = 1.37x^3 - 2.7$

12. (a)

x	−4	−3	−2	−1	0	1	2	3	4
y	11	9	7	5	3	−1	−2	−3	−4

(b) (i) –4

(ii) $2\frac{1}{2}$

(c) Gradient = –2

Revision Exercise 3.2 *(page 129–131)*

1. (a) x^2 : –1, 1 ; $-4x$: 8, 4, –4, –8, –12 ; y : 8, 2, 4

(c) $x = -1.95, 1.45, 2.4$

(d) (i) $x = -1.45, 0.4, 3.1$

(ii) $x = -1.4, 0.75, 2.6$

2. 1.44 cm

3. (a) 67.4°

(b) 78.2°

(c) 11.1 cm

4. (a) 55.2 cm

(b) 73.3°

(c) 16.7°

(d) 9.3°

5. 13.34°

6. (a) $2(a + b) \geq 80$, $a \leq 2b$

(b) $a - b \geq 3$, $a + b \leq 20$

8. (a) (i) 20

(ii) 33

(b) 16

9. $x + y > 3$, $y > \frac{1}{2}x$, $x \geq 0$, $y \leq \frac{-5}{8}x + 5$

10. (a) $2x^3$: –85.75, –16, –10.72, –6.75, –2, 6.75

(c) $x = 0.70$

11. (a) $x = 0$, $y = 2$

(b) $x = 3$, $y = 2$

12. (a) (i) 9.3

(ii) 0

(b) $x = -0.9$, $x = 1.2$, $x = 3.8$ (1dp)

Revision Exercise 3.3 *(page 131–132)*

1. (a)

x	0	0.25	0.5	1.0	1.5	2	2.5	3
y	0	10.3	17.5	24	22.5	16	7.5	0

(c) (i) 0.425, 2.05

(ii) $0 \leq x \leq 0.5$ and $1.9 \leq x \leq 3$

(iii) 110

2. 35.26°

3. (b) (i) 1.4 cm

(ii) 120.32 cm^2

4. (a) 22.41 cm

(b) 34.69°

(c) 73.04°

5. (a) 4.5 cm

(b) 73.3°

6. Vertices (0.9, 3.8), (6, 0), (16, 0), (1.85, 5.65)

Maximum value of $x + y$ is 15.9

7. (a) $x + y \leq 40$, $10x + 20y \leq 500$, $y \leq 15$

(c) 20 cars, 15 buses

(d) K 4 000

8. (a) $\frac{5}{4}$

(b) 7

9. $y - x < 2$, $4y \geq x - 1$, $y \leq x + 4$

10. 3

11. (a) 96 m

(b) 104 m

(c) 3.2 seconds

12. (b) x is inversely propotional to y

(c) 0.05

MODEL PAPERS

SET I PAPER I ANSWERS

(Student's Book pages 133–135)

1. (a) $x^2 - x - 2 = 0$

(b) $12x^2 + x - 6 = 0$

2. $\angle PRQ = 53.13^\circ$

3. Distance of B from N is 65.2 m

4. Total surface area = 43.81 cm^2

5. Distance = 25.98 km

6. x' (4, 4) y' (8, 4) z' (6, 8)

7. Two possible areas: 22 cm^2 and 352 cm^2
Two possible volumes: 6 cm^3 and 384 cm^3

8. Midpoint of CD: (–4, –0.5)
Position vector of midpoint of AC: $\begin{pmatrix} -3 \\ -2.5 \end{pmatrix}$

9. See graph on the next page.

10. $y = \frac{112}{3x} + \frac{5}{3}$, $y = 120$

11. (a) (i) –10

(ii) $\frac{2}{3}$

(b) $n = \frac{\log y - 1}{\log x}$

12. Length of BC = $\sqrt{9 - 4\sqrt{5}}$ cm
Area $= \frac{1}{2}\{(\sqrt{5} + 2)(\sqrt{9 - 4\sqrt{5}})\}$

13. (a) Mean mass = 45.25 kg

(b) Standard deviation = 9.653

14.

15. (a) Initial speed = 1 m/s

(b) Acceleration = 1 m/s^2

(c) Distance travelled = 17.5 m

16. (a) 41.41°

(b) 952.4 m^3

(c) 15.41°

17. AC = 8 cm long

18. When $x = -2\sqrt{2}$, $y = 2\sqrt{2}$

When $x = -2\sqrt{2}$, $y = {}^{-}2\sqrt{2}$

19. $x = 78°$

20. (a) (i) $\sqrt{3}$

(ii) $2 + \sqrt{5}$

(iii) $\sqrt{3} - 3$

(iv) $2\sqrt{2} - 3\sqrt{3}$

(b) 0.828 4

MODEL PAPERS

SET I PAPER II ANSWERS

(Student's Book pages 136–138)

SECTION A

1. $(x + 1)(2x - 15)$

2. $3\frac{1}{2}$

3. $x = 4$

4. 2.9

5. $\frac{8}{27}$

6. $x = 5$

7. 67.72

8. $x = 1\frac{1}{3}$

9. 5.731 9

10. $\frac{-2}{3}$

11. $y \geq x + 3$, $y \geq 4$, $y \geq -2$

12. 2:3

13. 12 notes

14. x = 1 or 4 y = 4 or 1

15. $\sqrt{5} - \sqrt{3}$

SECTION B

16. (a) $12x^2 - 54x = 0$, $x = 4\frac{1}{2}$

(b) 26 cm by $2\frac{1}{2}$ cm

(c) 200%

17. (a) $\frac{4}{15}$

(b) $\frac{2}{15}$

(c) $\frac{1}{75}$

(d) $\frac{44}{75}$

18. (a) D(4, –4) T(2, 2)

(b) $180^{\circ} - 2a$

(c) AC = 6.3 cm, 12.7 cm (Idp)

(d) 40 sq units

19. (a) 100.4 cm^2

(b) 31.42 cm^2

20. (a) $x + y \leq 15$
$52x + 32y \geq 500$
$200x + 300y \geq 3\,500$
$x \geq 0$; $y \geq 0$

(b) 13

(c) K 82 800

21. (a) 60

(b) 54

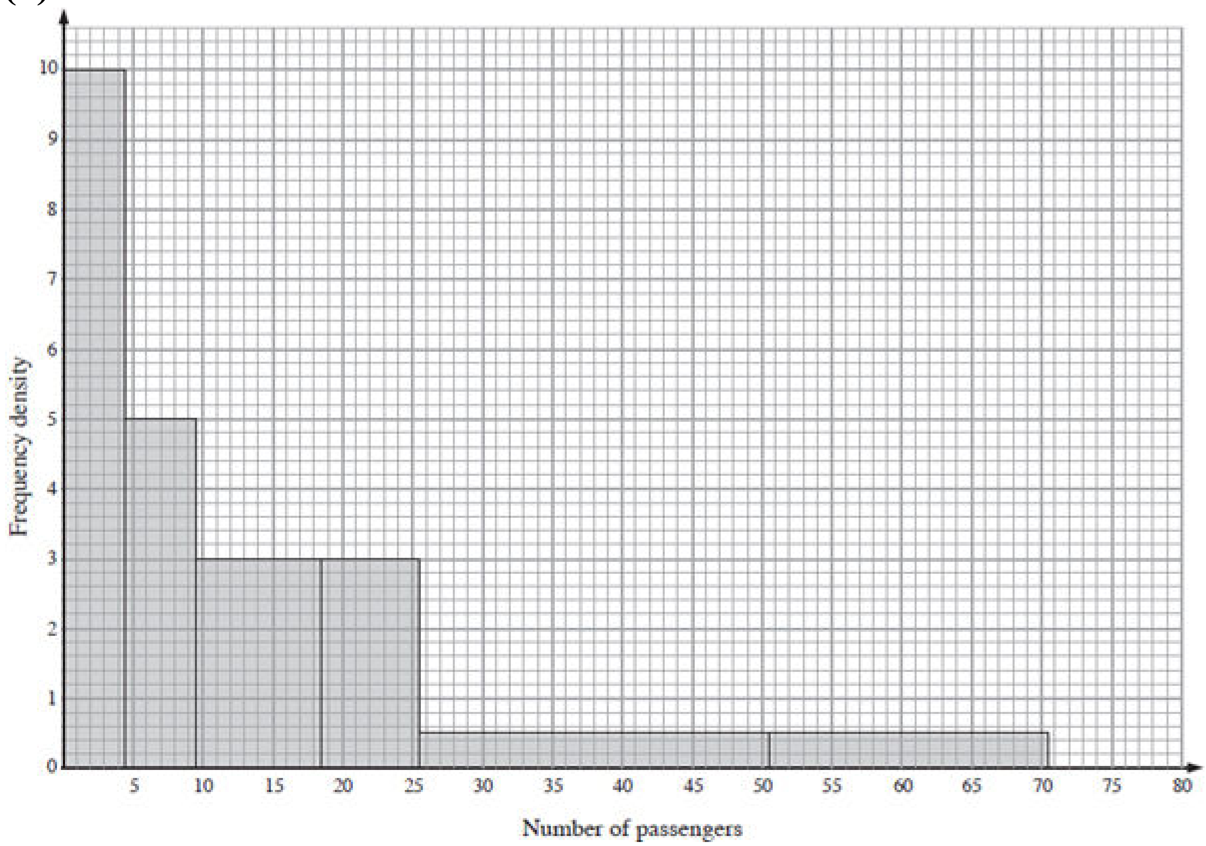
(c) 16

22. (b) 13.2 m

(c) 22.9 m

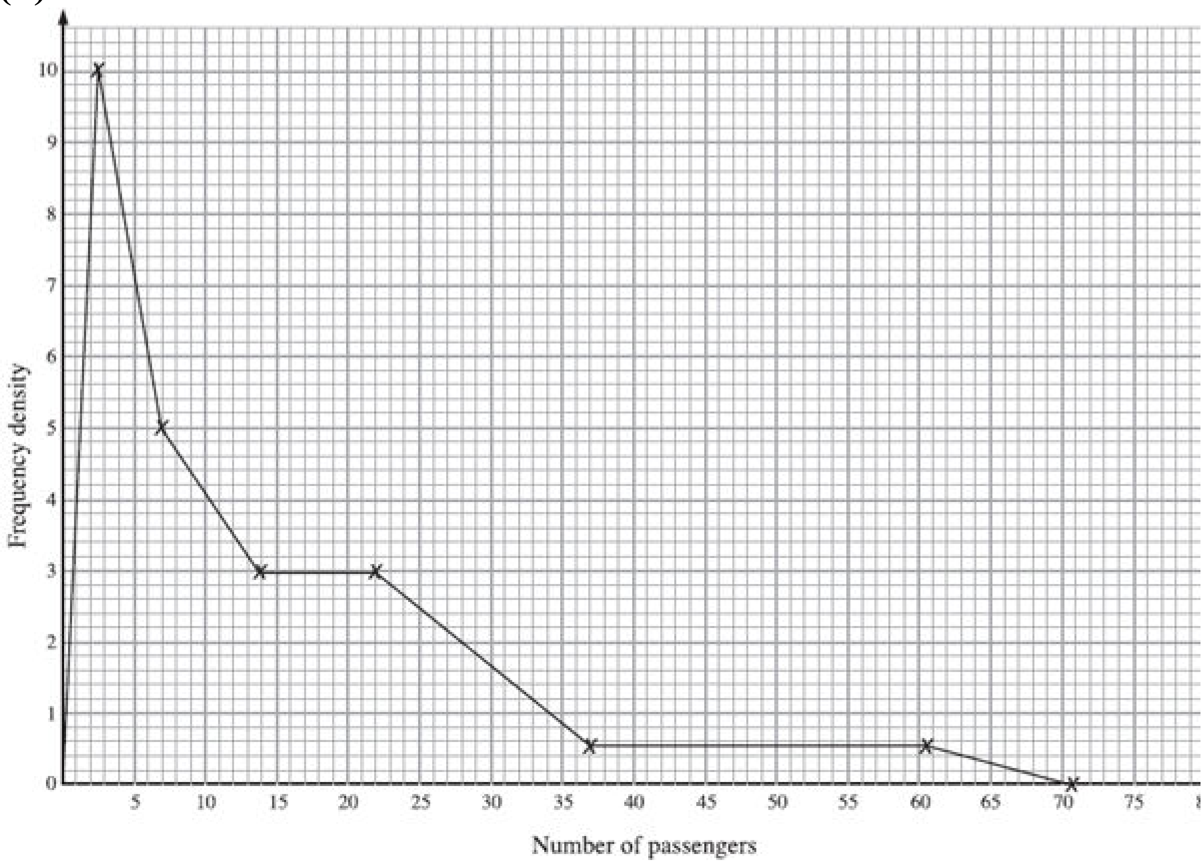
(d) 29.9 m

(a)



Frequency density
10
9
8
7
6
5
4
3
2
1
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
Number of passengers

(b)


Frequency density
0
1
2
3
4
5
6
7
8
9
10
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
Number of passengers

MODEL PAPERS

SET II PAPER I ANSWERS

(Student's Book pages 139–141)

1. (a) $\text{Sin}\ \theta = \frac{15}{17}$ $\text{Cos}\ \theta = \frac{8}{17}$

(b) $\text{Tan}\ \theta = \frac{3}{5}$

2. 24 cm

3. Centre (–5, 5) scale factor = 2

4. Gradient = $\frac{-5}{6}$

Equation of line is $y = \frac{-5}{6}x - \frac{-2}{3}$

5. $y = -2x + 13$

6. Distance travelled is 300 m

Average speed is 15 m/s

7. $\sqrt{5} = 2.24$

8. (a) 15 cm

(b) 15.71 cm

9. $x + y > 1$, $2x + 3y \leq 6$, $6 < 3/2x$, $y \geq 0$

10. (a) P (both will be alive) = 0.63

(b) P (neither will be alive) = 0.03

(c) P (neither will be alive) = 0.34

(d) P (atleast one will be alive) = 0.97

11. (a) $\underset{\sim}{B} = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$

(b) P = 6 q = –2

12. $x = 5$

13. $x = \frac{1}{2}, \frac{-1}{3}, 4$

14. When $x = 4$, $y = 3$
When $x = 3$, $y = 4$

15. $x = 0.27$ or -3.77

16. Table of values:

x	-3	-2	-1	0	1	2	3	4
y	-30	0	12	12	6	0	0	12

$x = \frac{1}{2}, \ -2, \ 3$

17. 324

18. 0.999 0

19. (a) 22.29 cm^2

(b) 20.11 cm^2

(c) 158 cm^2

20. (a) AB, BF, CG, CD

(b) BF, BC, CG, CD

(c) A, H, G, B

(d) (i) AC

(ii) BC

MODEL PAPERS
SET II PAPER II ANSWERS

(Student's Book pages 142–144)

SECTION A

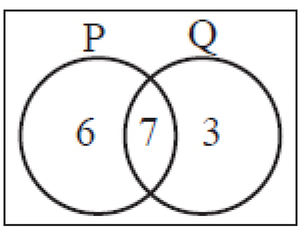
1. $n = \dfrac{9m - p^2}{mp^2 + 9}$

2. 2.50

3. (a) -2

(b) 29

4.



$n(\text{P}n\text{Q})$

5. (a) $v = 21$

(b) A = 27

6. AB = 7.75

7. (a) $\frac{1}{22}$

(b) $\frac{21}{44}$

8. (a) $\sqrt{2}$

(b) $\dfrac{3 - \sqrt{3}}{2}$

9. $y = 2$

10. (a) $x = 1$ or -2

(b) 13 and 15 or –13 and –15

11. (i) 80 km/h

(ii) 75 km/h

(iii) 16:15

12. 3 478.2

SECTION B

13. (a) $12x^2 - 54x = 0$, $x = 4$

(b) P = 6

(c) (i) $x = \pm 1$

(ii) $y = \pm 1$

14. (a) 262.81 cm^2

(b) 315.3 cm^3

15. (a) (i) 57.74 m

(ii) 123.8 m

(iii) 47.04 m

(b) 7 rolls

16. (a) (ii) 4

(b) (ii) (I) 386.02 cm^2

(II) 516.14 cm^3

17. (a) $x - 3.7$ or 1.2

(b) $x = -1.25$

18. (A) (a) 49:95

(b) 24 cm

(B) $x = 2$, $x = 2$, $x = \frac{-1}{2}$

19. (a) (i) 1.25 m/s^2

(ii) 0 m/s^2

(iii) –3.75 m/s^2

(c) 330 m

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